A Simplified, Analytical, One-Dimensional Model for Saturation Operation of the Bipolar Transistor

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ABSTRACT

A simple analytical model is given for saturation operation of the bipolar transistor. The model is valid for high-level injection and includes majority, as well as minority, currents.

1. INTRODUCTION

In saturation operation of the bipolar transistor there is a large base current, high minority carrier injection, substantial stored charge and low gain. Thermal equilibrium of majority carriers no longer exists and the simple approximation that majority carrier current is zero, used to describe operation in the high-gain active region, is no longer valid.

In order to construct a simple, one-dimensional, analytical model some means must be found to incorporate the lateral injection of base current into the active regions of the device. This can be done by use of an appropriate source function. For purposes of numerical analysis it has been assumed that base current flows uniformly throughout the base width.(1, 2) This leads to the use of a constant for the source function. However, a more intuitively acceptable description is to assume that the lateral base current, hence the source function, is proportional at all places to the majority carrier concentration. In this way the physical mechanism of majority carrier lateral current-flow can be incorporated into the model.

2. THEORY

The analysis which follows refers to an NPN device operating under high-level injection conditions with base stretching across to the collector-substrate boundary. Both minority and majority carrier currents are the sum of drfit and diffusion components. Divergence of minority current occurs by recombination; divergence of majority current occurs by recombination together with lateral injection. Thus the current equations are :

$$J_n = n e \mu_n E + eD_n \frac{dn}{dx} \qquad (1)$$

$$J_p = p e \mu_p E - eD_p \frac{dp}{dx} \qquad (2)$$

For high-level injection it is justified to consider that the electron and hole densities are equal and that both are much greater than the metallurgical doping. The neutrality equation is therefore very simple :

$$n = p \qquad (3)$$

For this case the Shockley-Hall-Read recombination mechanism reduces to $r = 1/(\tau_n + \tau_p)$ so the divergence of minority (electron) current can be expressed.

$$\frac{dJ_n}{dx} = ern \qquad (4)$$

The divergence of majority (hole) current can be expressed :

$$\frac{dJ_p}{dx} = egp - erp \qquad (5)$$

In this equation the coefficient 'g' is a rate constant which describes the lateral injection of base current. Thus :

$$J_B = eg \int_0^W p \, dx \qquad (6)$$

represents the total base current.

Divergence of base current is due to that part of the injected current which flows towards and leaves from the emitter or collector junctions. Thus :

$$J_{BR} = er \int_0^W p \, dx \qquad (7)$$

represents the component of base current due to recombination, and,

$$J_{BJ} = J_B - J_{BR} \qquad (8)$$

represents the component of base current due to the flow of holes across the emitter and collector junctions.

The solution of these equations proceeds as follows. substitute eqn. (3) into eqn. (2) then subtract from eqn. (1) to get :

$$2kT\,\frac{dn}{dx} = \frac{J_n}{\mu_n} - \frac{J_p}{\mu_p} \qquad (9)$$

Now differentiate (9) and Substitute (4) and (5) to get :

$$2kT\,\frac{d^2n}{dx^2} = -e\left(\frac{g}{\mu_p} - \frac{r}{\mu_p} - \frac{r}{\mu_n}\right) n \qquad (10)$$

which can be written :

$$\frac{d^2n}{dx^2} = -\beta^2 n \qquad (11)$$

where :

$$\beta^2 = \frac{e}{2kT}\left(\frac{g}{\mu_p} - \frac{r}{\mu_p} - \frac{r}{\mu_n}\right) \qquad (12)$$

The minus sign is used in equation (10) because in saturation operation the major component of base current is hole flow across the emitter and collector junctions. Thus $g > r$ and expressed as above the quantity β^2 is positive.

The solution of equation (11) is :

$$n = n_M \sin(\beta x + \phi) \qquad (13)$$

where n_M and ϕ are constants of integration.

This is a simple result and by substitution into the appropriate basic equations it is possible to obtain all quantities of interest to describe the operation of the transistor.

Notwithstanding the simplicity of the result expressed by eqn. (13) it is by no means a straightforward matter to obtain the values of the integration constants n_M and ϕ .

As a first step it is noted that at emitter and collector junctions :

$$n_B(o) = n_M \sin \phi \tag{14}$$

$$n_B(w) = n_M \sin (\beta w + \phi) \tag{15}$$

where the subscript 'B' is now used to denote values in the "stretched" base region of the device.

Now although the carrier distributions in the base region are not in thermal equilibrium it is still justified to consider that the carrier densities individually remain in thermal equilibrium across the emitter and collector junctions themselves. Thus :

$$n_B(o) = N_E \exp \left(-\frac{eV_{EJ}}{kT}\right) \tag{16}$$

where V_{EJ} is the internal barrier height at the emitter junction, and

$$n_B(w) = N_C \exp\left(-\frac{eV_{CJ}}{kT}\right) \tag{17}$$

The hole density injected into the emitter is

$$p_E(o) = p_B(o) \exp \left(-\frac{eV_{EJ}}{kT}\right) \tag{18}$$

and into the collector is

$$p_C(o) = p_B(w) \exp \left(-\frac{eV_{CJ}}{kT}\right) \tag{19}$$

The electron and hole densities in the base are equal and in particular $n_B(o) = p_B(o)$ and $n_B(w) = p_B(w)$. Thus :

$$p_E(o) = n^2{}_B(o) / N_E \tag{20}$$

$$p_C(o) = n^2{}_B(w) / N_C \tag{21}$$

These results for $p_E(o)$ and $p_C(o)$ can be used to derive the components of base current. Before doing so however the correction for band-gap narrowing should be applied. Thus :

$$p_E(o) = \frac{n_B^2(o)}{N_E} \exp\left(\frac{e\,\Delta V_{GE}}{kT}\right) \qquad (22)$$

$$p_C(o) = \frac{n_B^2(w)}{N_C} \exp\left(\frac{e\,\Delta V_{GC}}{kT}\right) \qquad (23)$$

The component of base current injected into the emitter is :

$$\left|J_{BE}\right| = \frac{e\,D_E\;p_E(o)}{L_E} \qquad (24)$$

and into the collector is

$$\left|J_{BC}\right| = \frac{e\,D_C\;p_C(o)}{L_C} \qquad (25)$$

where D_E, D_C represent the diffusion coefficients of holes in the heavily doped emitter and collector regions respectively, and L_E, L_C represent the diffusion lengths of holes in these regions being determined largely by Auger recombination.

The total base current is :

$$\left|J_B\right| = \left|J_{BE}\right| + \left|J_{BC}\right| + \left|J_{BR}\right| \qquad (26)$$

By use of these various equations it is possible to set up a suitable solution procedure.

Once the integration constants n_M and ϕ, together with the appropriate value of the source coefficient g, have been obtained, the behaviour and characteristics of the device can be described.

The hole current in the base can be evaluated from equations (5) and (20) and is :

$$J_p = -\frac{e\,D_E\;p_E(o)}{L_E} + \frac{e\,n_M}{\beta}(g-r)[\cos\phi - \cos(\beta x+\phi)] \qquad (27)$$

The electron current in the base can be evaluated from equations (9) and (27) and is :

$$J_n = 2\mu_n kT\beta\, n_M \cos(\beta x + \phi) + \frac{\mu_n J_p}{\mu_p} \qquad (28)$$

In particular, the collector current is the algebraic sum of the electron and hole currents across the collector junction and is :

$$J_C = 2\mu_n kT\beta\, n_M \cos(\beta w + \phi) + (1 + \frac{\mu_n}{\mu_p}) \frac{eD_C p_C(o)}{L_C} \qquad (29)$$

The collector-emitter voltage is the total potential difference between emitter and collector. It is the algebraic sum of the emitter junction potential step, the collector junction potential step and the integrated electric field across the base :

$$V_{EC} = (V_{ED} - V_{EJ}) - (V_{CD} - V_{CJ}) - \int_o^w E\, dx \qquad (30)$$

where V_{ED} and V_{CD} are the zero bias thermal equilibrium diffusion barrier heights at the emitter and collector junctions respectively.

$$V_{ED} = \frac{kT}{e} \ln (\frac{N_E N_B}{n_i^2}) \qquad (31)$$

$$V_{CD} = \frac{kT}{e} \ln (\frac{N_C N_B}{n_i^2}) \qquad (32)$$

Thus the collector voltage is given by :

$$V_{EC} = \frac{kT}{e} \ln [\frac{n_B(o)}{n_B(w)}] - \int_o^w E\, dx \qquad (33)$$

The electric field is given by equation(2) as :

$$E = \frac{J_n}{e\mu_n n} - \frac{kT}{en} \frac{dn}{dx} \qquad (34)$$

which is :

$$E = \left[\frac{g-r}{\mu_p \beta} \cos \phi - \frac{|J_{BE}|}{e\, \mu_p\, n_M} \right] \operatorname{cosec} (\beta x + \phi)$$

$$+ \left[\frac{\beta k T}{e} - \frac{g - r}{\mu_p \quad \beta} \right] \cot (\beta x + \phi) \qquad (35)$$

The integrated field gives the voltage rise across the base from emitter to collector as :

$$V_{BASE} = A \ln \left[\frac{1 + \cos(\beta w + \phi)}{1 - \cos(\beta w + \phi)} \cdot \frac{1 - \cos \phi}{1 + \cos \phi} \right]$$

$$+ B \ln \left[\frac{\sin (\beta w + \phi)}{\sin \phi} \right] \qquad (36)$$

where :

$$A = \frac{g - r}{\mu_p \beta^2} \cos \phi - \frac{|J_{BE}|}{e\, \mu_p \beta\, n_M} \qquad (37)$$

$$B = \frac{kT}{e} - \frac{g - r}{\mu_p \beta^2} \qquad (38)$$

Thus the collector voltage is :

$$V_{EC} = \frac{kT}{e} \ln \left[\frac{n_B(o)}{n_B(w)} \right] + V_{BASE} \qquad (39)$$

The 'stored charge' in the base is given by :

$$Q_B = e \int_o^w n \, dx \qquad (40)$$

and is :

$$Q_B = e\, n_M \left(\cos \phi - \cos (\beta w + \phi) \right) / \beta \qquad (41)$$

The lateral biasing at the centre of the emitter can be estimated from the formula

$$V_T = R_S J_B L^2 / 8 \tag{42}$$

where L is the emitter width and R_S is the base sheet resistivity given by :

$$R_S = 1 / \mu_p Q_B \tag{43}$$

Thus the emitter width for a given lateral bias at the centre is :

$$L = 2 (2\mu_p V_T Q_B / J_B)^{\frac{1}{2}} \tag{44}$$

This is valid provided $V_T << kT/e$ so that the current 'fall-off' due to lateral biasing is small and linear.

3. RESULTS

The foregoing analysis is applicable to any bipolar transistor which is operating under conditions of high-level injection such that the mobile carrier density in the base is much greater than the nett metallurgical doping density in the base.

For purposes of illustration the analysis is applied to a hypothetical power transistor with device constants as given in Table 1.

Table 1

Device Constants of power bipolar transistor

Base width W	3E-5
Electron mobility μ_n	5E-2
Hole mobility μ_p	2E-2
Diffusion coefficient of electrons in collector (substrate)	2.5E-4
Diffusion coefficient of holes in emitter	2.5E-4
Diffusion distance of electrons in collector (substrate)	3E-6
Diffusion distance of holes in emitter	3E-6
Band-gap narrowing ΔV_G	6E-2
Electron lifetime τ_n	5E-6
Hole lifetime τ_p	20E-6

Collector characteristics are plotted in Fig. 1 and show the low gains and small collector voltages which are typical for saturation operation. The gain characteristics are plotted in Fig. 2 and the "stored" charge characteristics in Fig. 3. Emitter widths, for 10% fall-off in current at the centre, are shown in Fig. 4.

4. CONCLUSIONS

A simple analytical, one-dimensional model has been given for the bipolar transistor in saturation operation. The model has yet to be tested against experiment but should be able to provide physical insight and design and analysis facilities for power transistors and integrated logic transistors.

5 ACKNOWLEDGEMENTS

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6. REFERENCES

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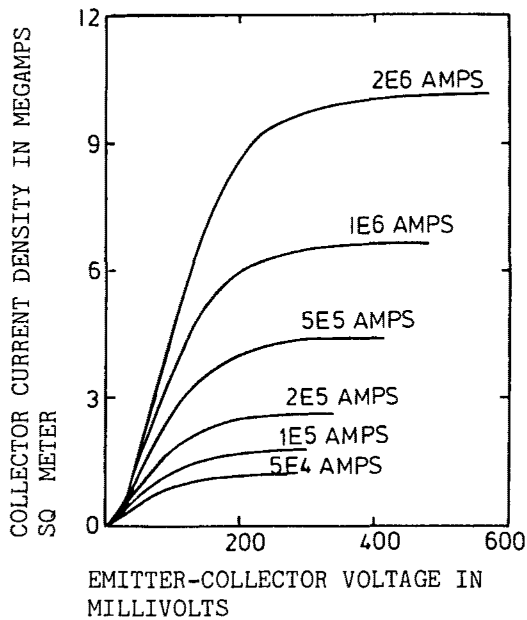

FIG. 1 Collector Characteristics of Bipolar Power Transistor. Saturation Operation - Base Drives as specified.

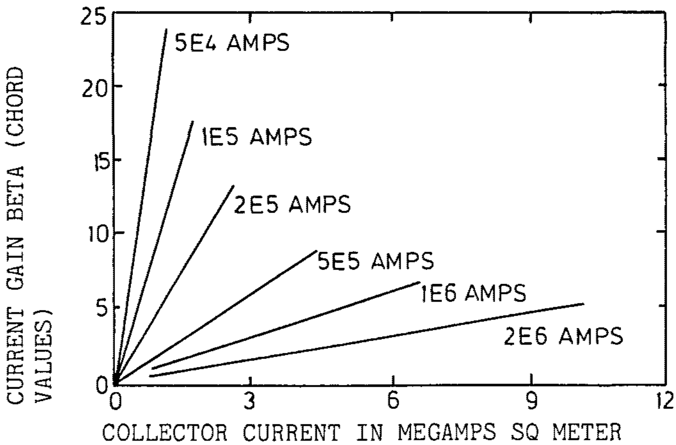

FIG. 2 Gain Characteristics of Bipolar Power Transistor.
Saturation Operation - Base Drives as Specified.

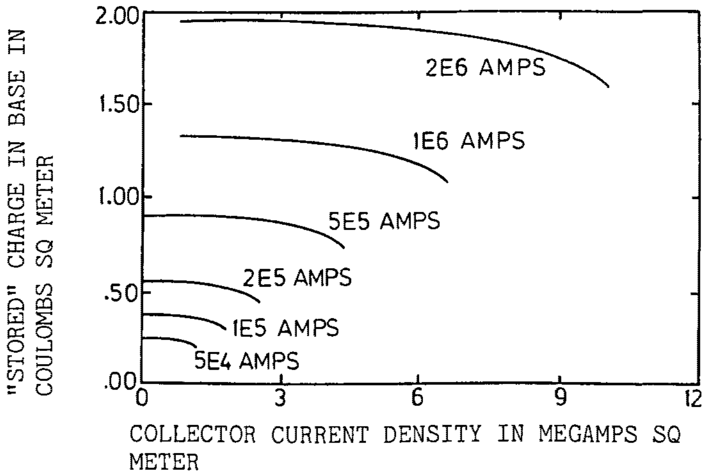

FIG. 3 "Stored" Charge Characteristics of Bipolar Power Transistor.
Saturation Operation - Base Drives as Specified.

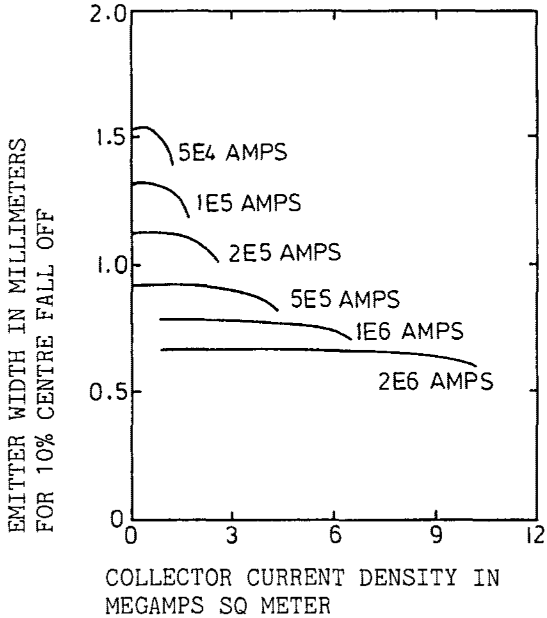

FIG. 4 Emitter Width Characteristics of Bipolar Power Transistor. Saturation Operation - Base Drives as Specified.