

Semiconductor Device Simulation using Adaptive Refinement and Streamline Upwinding

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Summary

An adaptive mesh refinement scheme and data structure has been developed in conjunction with a streamline upwind Petrov-Galerkin finite element formulation for analysis of the semiconductor device equations. The nonlinear electrostatic potential equation and convection dominated carrier current continuity equations are iteratively decoupled in the solution algorithm. Incremental continuation is employed to improve the nonlinear solution iteration and to produce an efficient and robust scheme. The adaptive refinement scheme also employs an element-by-element conjugate gradient solution algorithm which performs very efficiently on parallel and vector processors. Sample numerical results for MOSFET problems indicate the effectiveness of the Petrov-Galerkin method and demonstrate its' superiority over traditional Scharfetter-Gummel approaches.

Extended Abstract

Considerable effort has been directed recently to the development of numerical techniques for the solution of the semiconductor device equations. Most approaches employ finite difference or finite volume discretization techniques, but finite element techniques have also been employed more recently, although to a lesser extent. In the semiconductor device problem the solution can exhibit extreme layers, particularly at the junctions, and the drift term in the current continuity equations yields a strongly convective effect. This convective operator is known to generate oscillations in solutions on practical grids unless numerical dissipation is employed. Such oscillations are coupled back through the nonlinearities into the equations so that the solution schemes are frequently nonlinearly unstable. These difficulties have lead researchers to employ the well known Scharfetter-Gummel upwind strategy in previous studies.

Recent research in fluid mechanics has led to the development of streamline upwind Petrov-Galerkin techniques for finite element analysis. In this variant of the finite element method, numerical dissipation is introduced specifically in the flow (drift) direction. This idea has been incorporated in the present implementation for the current continuity equations. Numerical studies have been made to compare this with a standard Scharfetter-Gummel scheme. The new technique captures the layers in the potential and carrier concentrations more accurately than the standard method.

This scheme is implemented in conjunction with an adaptive mesh refinement algorithm for bilinear quadrilateral finite elements. The grid is adaptively refined in regions where solution gradients are high so that layers are better resolved. Hence the solution and mesh are improved successively during the solution process with error indicators for the adaptive refinement scheme computed from the current approximate solution, and so on. Since the solution on the previous grid provides an excellent starting iterate for the solution on the next grid, iterative methods for the linear subsystem are particularly attractive. Here we employ an element-by-element conjugate gradient iteration for the linear subsystems.

Since the operators for the current continuity equations are convection dominated, the resulting coefficient matrices will be nonsymmetric. To accommodate the asymmetry a bi-conjugate gradient (Lanczos) iterative scheme is employed. Rather than assemble the finite element system in the standard manner, we instead implement the bi-conjugate gradient iteration at the element level. There is then a very low storage demand and the simulation can be made on engineering work-stations or even personal computers. Alternatively, the scheme vectorizes easily over the number of elements in the mesh and the vector lengths are extremely long. This implies that the approach will be particularly effective on vector machines such as the CRAY and particularly the CYBER 205 or ETA. We have implemented a vectorized

form of the element-by-element algorithm on the CRAY X-MP vector processor. This approach can also be exploited in parallel processing. Finally, the Poisson and current continuity equations are decoupled iteratively in the solution algorithm and incremental continuation in applied bias is utilized to accelerate convergence.

An example calculation for a MOSFET yields the grid in Figure 1 after three refinement steps. The corresponding solution for electrostatic potential and carrier concentrations are shown in Figures 2 and 3. Note the density of the mesh in the regions where layers are formed. The calculations shown were made with the streamline upwinding strategy. As a comparison, the Scharfetter-Gummel solution for the electrostatic potential field alone is shown in Figure 4. Notice the sharper layer approximation in Figure 2 in contrast to the more dissipative solution in Figure 4. In both schemes, as the mesh is refined and local element sizes decrease, the amount of numerical dissipation is automatically reduced. This implies that on coarser grids there is more dissipation and this tends to stabilize coarse grid calculations as well as improve efficiency.

Conclusion

The use of adaptive refinement with "streamline upwind Petrov-Galerkin finite element solution" leads to a particularly accurate and robust scheme. Numerical results are superior to alternative schemes and the element-by-element conjugate gradient solution with incremental continuation in the boundary conditions proves very efficient.

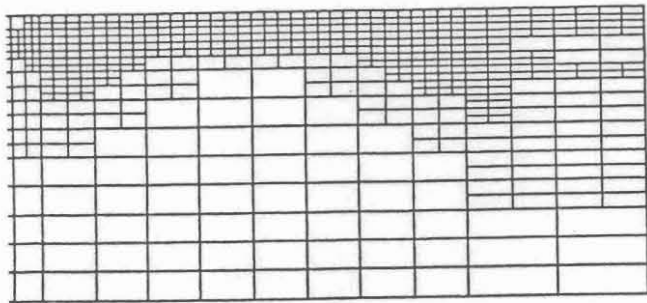


Figure 1

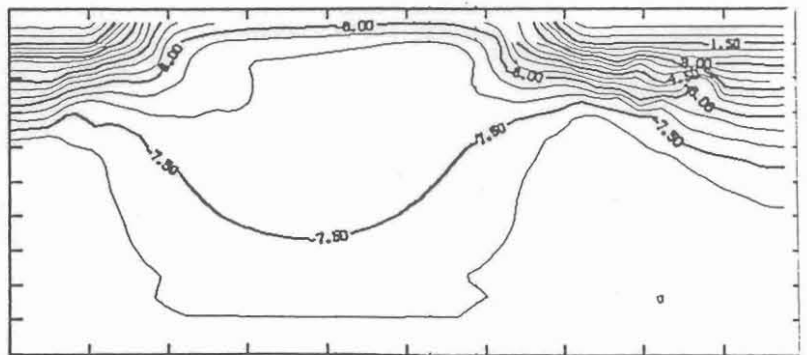


Figure 3

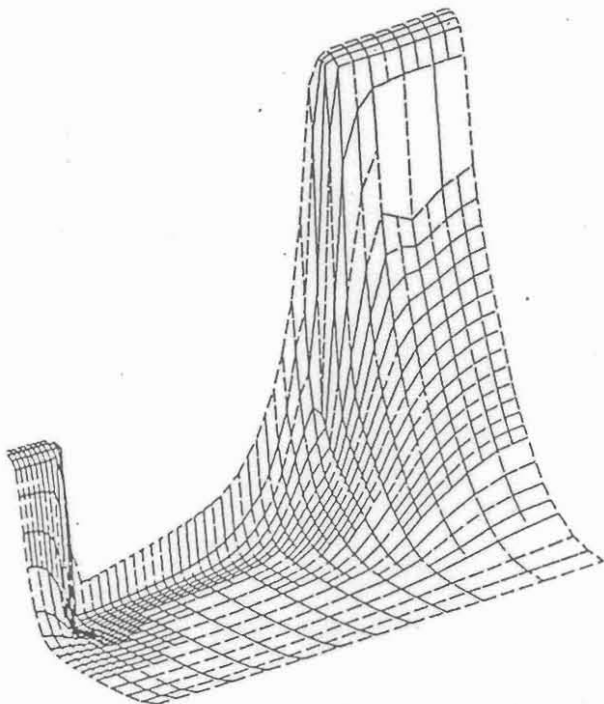


Figure 2

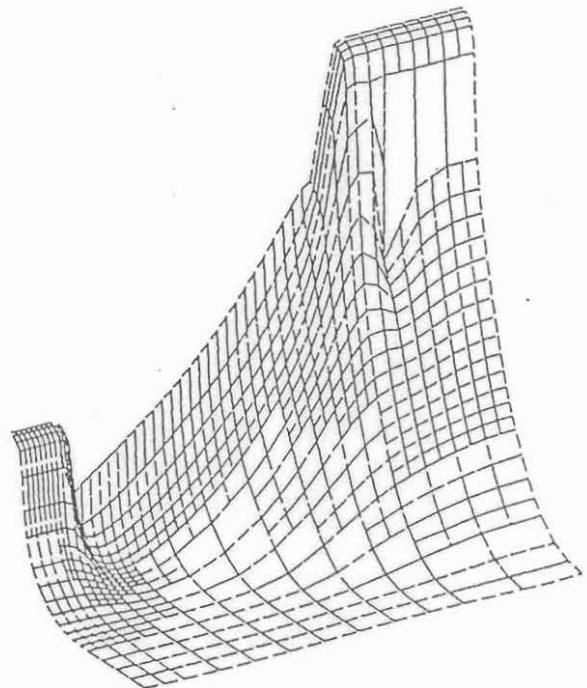


Figure 4