

**ON THE DISCRETIZATION OF THE HYDRODYNAMIC MODEL
OF THE SEMICONDUCTOR EQUATIONS**

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Summary: *In this paper we discuss a new discretization method for the inner product $\mathbf{E} \cdot \mathbf{J}_n$ which appears in the hydrodynamic model of the semiconductor equations. The obtained results are very satisfactory so long as the mesh is suitably refined in the hot spots of the simulated device. Thus, an adaptive refinement scheme accounting for electron-temperature variations turns out to be necessary.*

Abstract

In recent papers, a new discretization strategy of the semiconductor equations comprising momentum and energy balance has been proposed [1,2]. A major discretization problem still to be solved regards the inner product $\mathbf{E} \cdot \mathbf{J}_n$ [2], which represents the forcing term for carrier heating. In this paper we address the above problems by proposing a new discretized form for $\mathbf{E} \cdot \mathbf{J}_n$ and show that, by using a suitable adaptive mesh-refinement technique, smooth solutions can be achieved with a reasonable number of mesh points.

The proposed discretization scheme for $\mathbf{E} \cdot \mathbf{J}_n$ is based on the simple vector relationship

$$\mathbf{E} \cdot \mathbf{J}_n = -\text{div}(\varphi \mathbf{J}_n) + \varphi \text{div} \mathbf{J}_n \quad (1)$$

By remembering that, in steady-state, $\text{div} \mathbf{J}_n = qU$, U being the net recombination rate, eq. (1) leads to the following form of the energy-balance equation

$$\text{div}(\mathbf{S}_n + \varphi \mathbf{J}_n) = (q\varphi - w_n)U + n \left(\frac{\partial w_n}{\partial t} \right)_{\text{coll}} \quad (2)$$

where \mathbf{S}_n is the energy flow and w_n is the mean electron energy. The term on the LHS of (2) can be interpreted as the divergence of the *total* energy flow, while the first term on the RHS of (2) represents the total-energy density lost per unit time due to recombination.

The discretized form of $\mathbf{E} \cdot \mathbf{J}_n$ at the i th node turns out to be

$$-\sum_{j \neq i} \left[\frac{d_{ij}}{s_{ij}} J_{ij}^n \frac{(\varphi_i + \varphi_j)}{2} \right] + q\varphi_i U_i \Omega_i \quad (3)$$

where J_{ij}^n is the projection of the current density over the element side (i, j) , d_{ij} is its cross-section, s_{ij} is the length of the side (i, j) and Ω_i is the area of the i th box. It is worth mentioning that (3) does not involve the problem of computing the current density \mathbf{J}_n in the neighborhood of the i th node; rather all the physical parameters appearing in (3) are either nodal values of scalar quantities or projections of the current density over the sides emanating from the i th node.

Figure 1 shows a 3-D plot of the carrier temperature in a typical $1\ \mu\text{m}$ FET biased with $V_{GS} = 1\text{V}$ and $V_{DS} = 5\text{V}$. The same plot is shown from a different viewpoint in figure 2, which clearly exhibits a temperature ridge in the vicinity of the metallurgical drain junction. As can be seen, the electron temperature is rather smooth at this refinement level even around the temperature peak which occurs at the drain end of the channel. This confirms that the solution adopted for the discretization of $\mathbf{E} \cdot \mathbf{J}_n$ provides satisfactory results, so long as the mesh is properly refined in the neighborhood of the peak. On the other hand, due to the difficulty of *a priori* foreseeing the regions where intense refinement is required, an adaptive scheme based on temperature variation is necessary. Experiments are currently in progress in order to combine the solution of the semiconductor equations with such an adaptive refinement scheme.

References

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2. A. Forghieri, R. Guerrieri, P. Ciampolini, A. Gnudi, M. Rudan and G. Baccarani: "A New Discretization Strategy of the Semiconductor Equations Comprising Momentum and Energy Balance", *International Workshop on Numerical Modeling*, Santa Clara, Calif., 13-14 November 1986. Also, *IEEE Trans. on CAD of ICAS*, (to appear in February 1986).

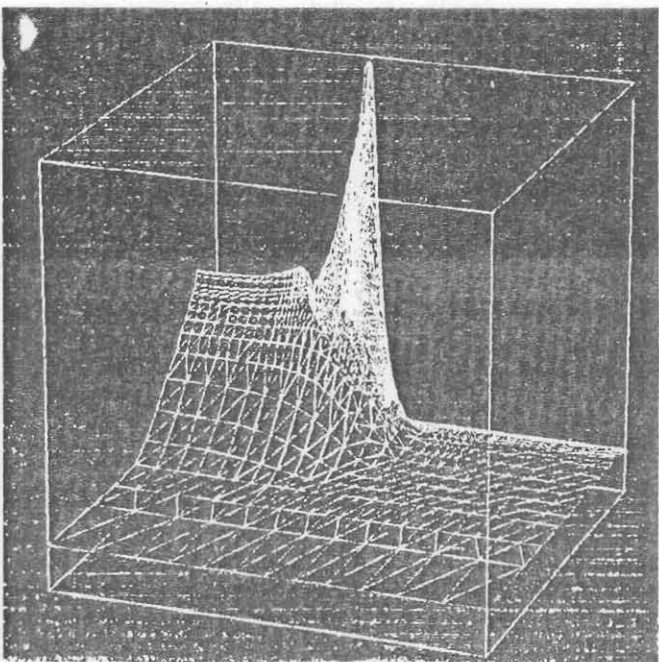


Fig. 1 : electron temperature

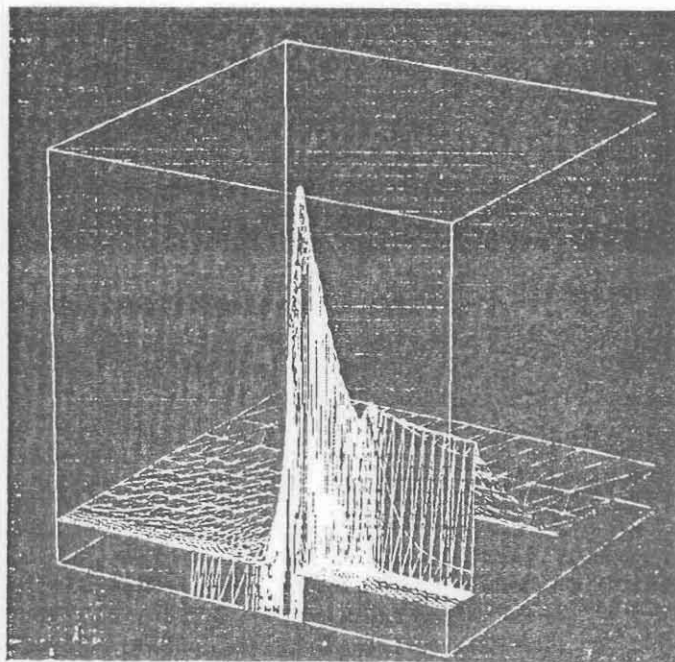


Fig. 2 : electron temperature