Numerical Methods for the Hydrodynamic Device Model

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Abstract. Numerical simulations of the ballistic diode using the hydrodynamic device model are presented. The stability, convergence, and accuracy of various numerical methods are analyzed. Special attention is given to the proper choice of state variables.

The hydrodynamic model [1] plays an important role in simulating the behavior of charge carriers in submicron semiconductor devices, since it exhibits velocity overshoot and ballistic effects missing in the drift-diffusion model. We present the results of simulations of the 1D ballistic diode based on the hydrodynamic device model, and compare our results (based on different numerical methods) with those of Odeh, Rudan, and White [2].

For the model ballistic diode problem, the effects of holes may be neglected. The hydrodynamic model in one dimension then consists of three nonlinear conservation laws (for the density, momentum density, and energy density of electrons), plus Poisson's equation for the electric potential.

We use state-of-the-art numerical methods, and survey the effects of different (1) discretizations (central difference, Scharfetter-Gummel, upwind), (2) block splittings of the system of equations, (3) ways of solving the linearized device equations (direct vs. iterative solutions), and (4) choices of variables.

We analyze the properties of the Scharfetter-Gummel discretization in the light of upwind methods, and discuss the applicability of transport-diffusion algorithms [3] to the hydrodynamic model.

Next we analyze the rate of convergence in solving the full set of hydrodynamic equations vs. various block splittings.

We use a two-parameter damped Newton method [4] to linearize the device equations. We then compare direct and iterative methods for solving these linearized equations. In the iterative method case, we discuss how to optimize the number of inner iterative loops vs. outer Newton loops.

Finally we compare results using different choices of state variables: e.g. the conserved quantities density, momentum density, and energy density, vs. density, velocity, and temperature.

References

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