Analysis and Simulation of Basic Memristor Properties

Bernardo Tellini[1], Mauro Bologna[2], and Kristopher J. Chandia[2], Massimo Macucci[3]
[1] *Dipartimento di Ingegneria dell'Energia, dei Sistemi, del Territorio e delle Costruzioni, University of Pisa, Pisa, Italy*
[2] *Departamento de Ingeniería Eléctrica – Electrónica, Universidad de Tarapacá, Arica, Chile*
Dipartimento di Ingegneria dell'Informazione, University of Pisa, Pisa, Italy
macucci@mercurio.iet.unipi.it

We present an analysis of the memristor concept first defined by Chua in 1971 [1], pointing out that the flux linkage appearing in its definition cannot be considered to be equivalent to the actual magnetic flux (as claimed in [1]). The flux linkage is just the time integral of the voltage and coincides with the magnetic flux only in the case of an inductor. Once this fact is recognized, it is straightforward to show that the memristor is a dissipative element and that it is just an extension of the resistor concept: a resistor whose value evolves in time according to a set of state variables. In the original formulation, one of the defining properties of a memristor was that it established a single-valued relationship between the flux linkage φ and the charge q: we show that this is guaranteed only for the most basic memristors, i.e. those in which the resistance depends only on the charge. As an example, we perform a numerical simulation of a particular memristor: a thermistor, whose resistance is a decreasing function of temperature. In Fig. 1 we report the behavior in the *V-I* plane of such a thermistor for a very low frequency cyclic excitation. In Fig. 2, we present the corresponding curve in the φ-q plane: it is single-valued because of the very low frequency (leading to a "steady-state" thermal condition), but as the frequency is raised (and becomes comparable with the reciprocal of the thermal time constant), a hysteretic behavior appears both in the *V-I* (Fig. 3) and in the φ-q (Fig. 4) plane. In this specific case, the φ-q curve is not single-valued, but it is at least closed (as a result of a particular symmetry in the *V-I* curve). In order to further support our conclusion that the memristor is just an extension of the resistor concept, we present a simple memristor model, consisting of resistors and charge-controlled switches. The schematic diagram is reported in the top panel of Fig. 5, while in the bottom panel we show its behavior in the *V-I* plane. As for all memristors, the curve in the *V-I* plane is pinched in the origin. In Fig. 6 we report the corresponding curve in the φ-q plane, which, consistently with the original definition, is a single-valued curve.
After discussing some other examples, we remark that the requirement that the *V-I* curve be pinched in the origin is an expression of the fact that no energy can be stored and that the memristor is a purely dissipative element. Furthermore, a direct consequence of our analysis is that the memristor is not a fourth basic circuit element.

[1] L. O. Chua, IEEE Trans. Circuit Theory, **CT-18**, n. 5, 507 (1971)

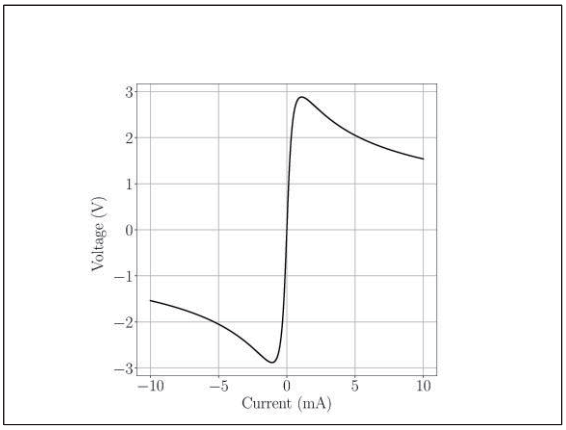

Fig.1: Curve in the V-I plane for a thermistor operated at a very low frequency with a cyclic current excitation.

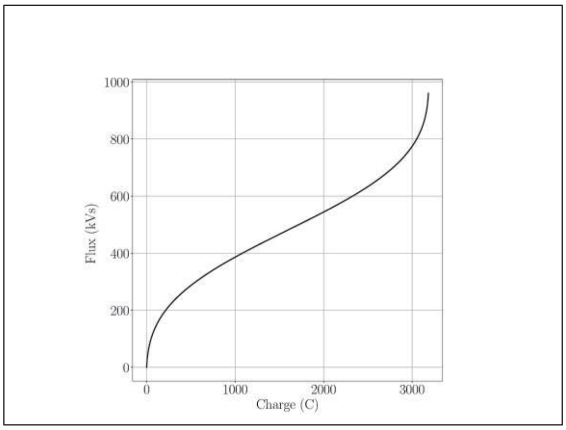

Fig.2: Curve in the φ-q plane for a thermistor operated at very low frequency.

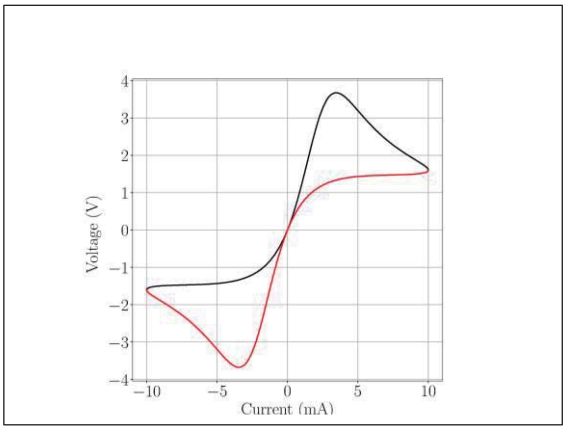

Fig.3: Curve in the φ-q plane for a thermistor operated at a frequency comparable to the reciprocal of the main time constants characteristic of the device.

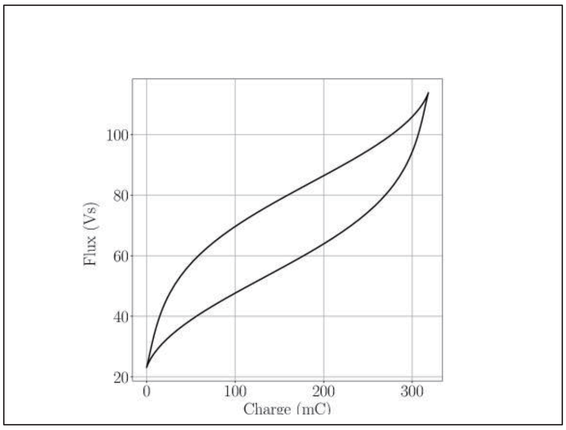

Fig.4: Curve in the φ-q plane for a thermistor operated at a frequency comparable to the reciprocal of the time constants characteristic of the device.

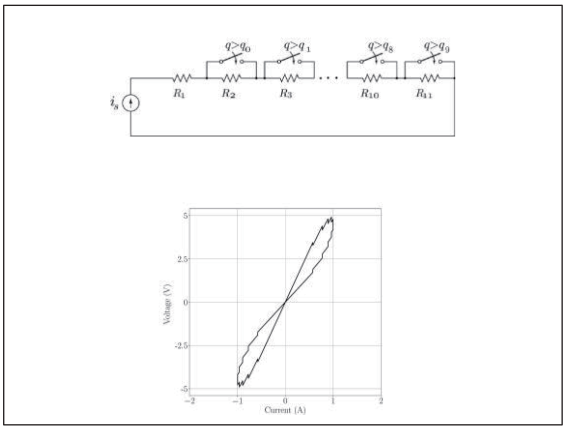

Fig.5: Top panel: memristor model based on resistors and charge-controlled switches; Bottom panel: Curve in the V-I plane for the memristor based on resistors and charge-controlled switches.

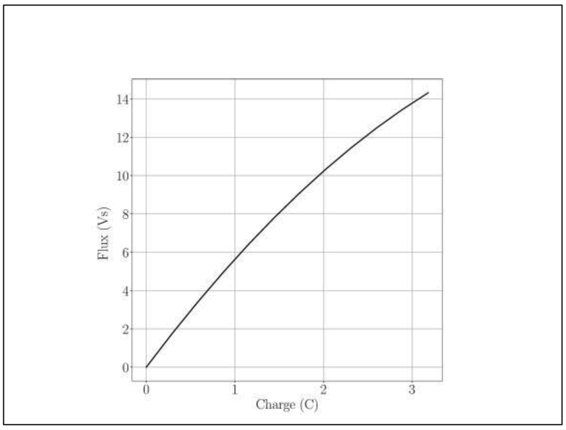

Fig.6: Single-valued curve in the φ-q plane for our memristor model based on resistors and charge-controlled switches.