Assessing quantum thermalization in small isolated quantum systems through local-in-position weak values of the momentum

Carlos F. Destefani[1], Guillermo Albareda[2], and Xavier Oriols[1]

[1] *Universitat Autònoma de Barcelona, Barcelona, Spain*
[2] *Max Planck Institute for the Structure and Dynamics of Matter, Hamburg, Germany*
Carlos.Destefani@uab.cat

The study of thermal equilibrium in isolated quantum systems is nowadays among the most intriguing and fundamental problems in statistical mechanics [1]. In the literature, the understanding of thermalization usually differentiates between two types of systems: large isolated systems, where equilibration is reached only by their macroscopic properties, and small isolated systems, where their microscopic properties themselves reach equilibrium values [2]. The later type of thermalization has been experimentally demonstrated for a system with as few as 6 degrees of freedom [3].

The question of how information of a small and isolated quantum system can be extracted without perturbing it appears as a challenging technological and even fundamental (due to contextuality) problem. In this respect, there is large theoretical and experimental interests on weak values because they provide information on quantum systems beyond what is obtained from conventional expectation values. Weak value thus arises as the only tool that can provide dynamical information of a closed system with few degrees of freedom without perturbing it (with zero back-action).

In this work we show that local-in-position weak values of the momentum are an advantageous tool to study quantum thermalization in isolated quantum systems with few degrees of freedom. Such local-in-position weak values have already been experimentally measured in the laboratory and their prediction, from the theoretical viewpoint, just require a Bohmian (hydrodynamic) formulation of the dynamics of the isolated quantum system [4,5].

As an example, here, we consider the quantum dynamics of a coherent state in a one-dimensional harmonic oscillator under the presence of different amplitudes and correlation lengths of a random Gaussian disorder potential [6]. Fig. 1 shows a typical situation: harmonic potential plus a small amplitude disordered potential (left panel), initial and final wavefunctions within the classical turning points (right panel). By solving the time-dependent Schrodinger equation we first discuss a particular criterion for assessing the disorder-assisted equilibration, which is based on the analysis of the kinetic and potential energies alongside the conservation of the total energy (Fig. 2, top panel). The Bohmian (hydrodynamic) formulation of the problem provides information on the local-in-position weak values of the momentum (Bohmian velocities), showing that equilibration in this simple example corresponds to final quantum trajectories localized at certain real-space configurations (vanishing Bohmian velocities) as if they were associated to a stationary state (Fig. 2, bottom panel).

The great merit of the novel proposed way of understanding thermalization in terms of local-in-position weak values (Bohmian velocities) is that such dynamic information is available experimentally [7], ensuring no contamination (zero back-action) from the measuring apparatus [5].

[1] C. Gogolin and J. Eisert, Rep. Prog. Phys. **79**, 056001 (2016).
[2] G. Albareda, A. Riera, M. González, J. M. Bofill, I. P. R. Moreira, R. Valero, and I. Tavernelli, Phys. Chem. Chem. Phys. **22**, 22332 (2020).
[3] A. M. Kaufman, et al., Science **353**, Issue 6301, 794 (2016).
[4] H. M. Wiseman, New J. Phys. **9**, 165 (2007).
[5] F. L. Traversa, G. Albareda, M. Di Ventra, and X. Oriols, Phys. Rev. A **87**, 052124 (2013)
[6] Y.-W. Hsueh, C.-H. Hsueh, and W.-C. Wu, Entropy **22**, 855 (2020).
[7] D. Marian, N. Zanghì, and X. Oriols, Phys. Rev. Lett. **116**, 110404 (2016).

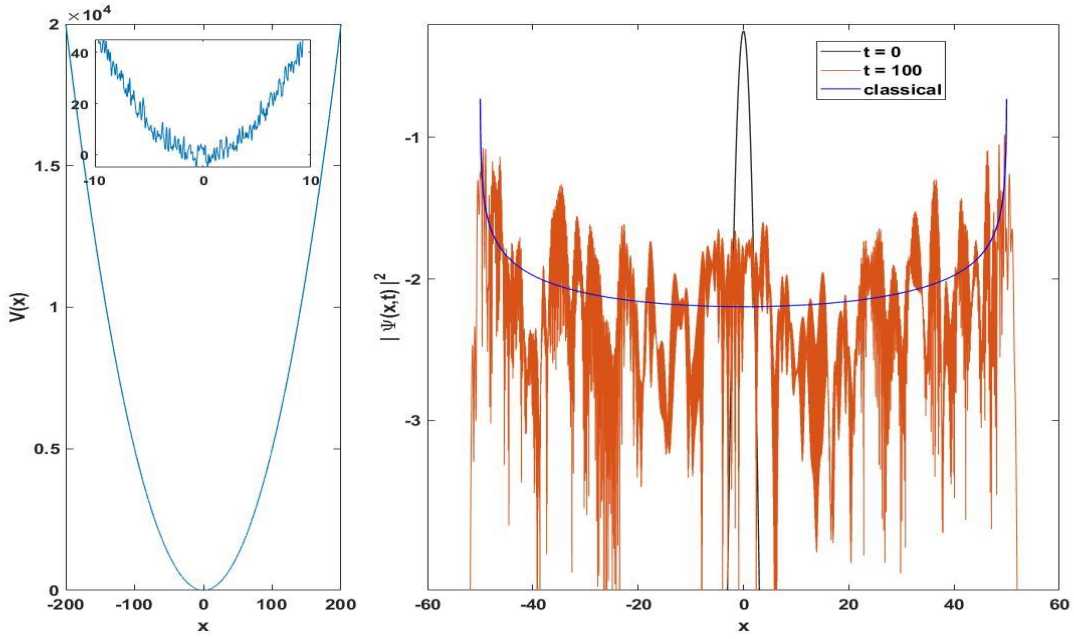

Fig. 1. *Left Panel: Harmonic potential data: x-grid [-64π,64π], k-grid: [-96,96], N=12288 points, resolutions dx=0.0327 and dk=0.0156; inset: zoom showing the random disorder with amplitude V_D= 50 and correlation length σ_D= 0.01. Right Panel (logarithmic scale in vertical axis): Gaussian initial wavefunction (black) at t=0, with initial velocity v=50; final wavefunction (red) at time t=100, with time step dt=0.0001; classical distribution (blue)* $1/\pi\sqrt{v^2-x^2}$*, with turning points at x=±50. Atomic units are implicit.*

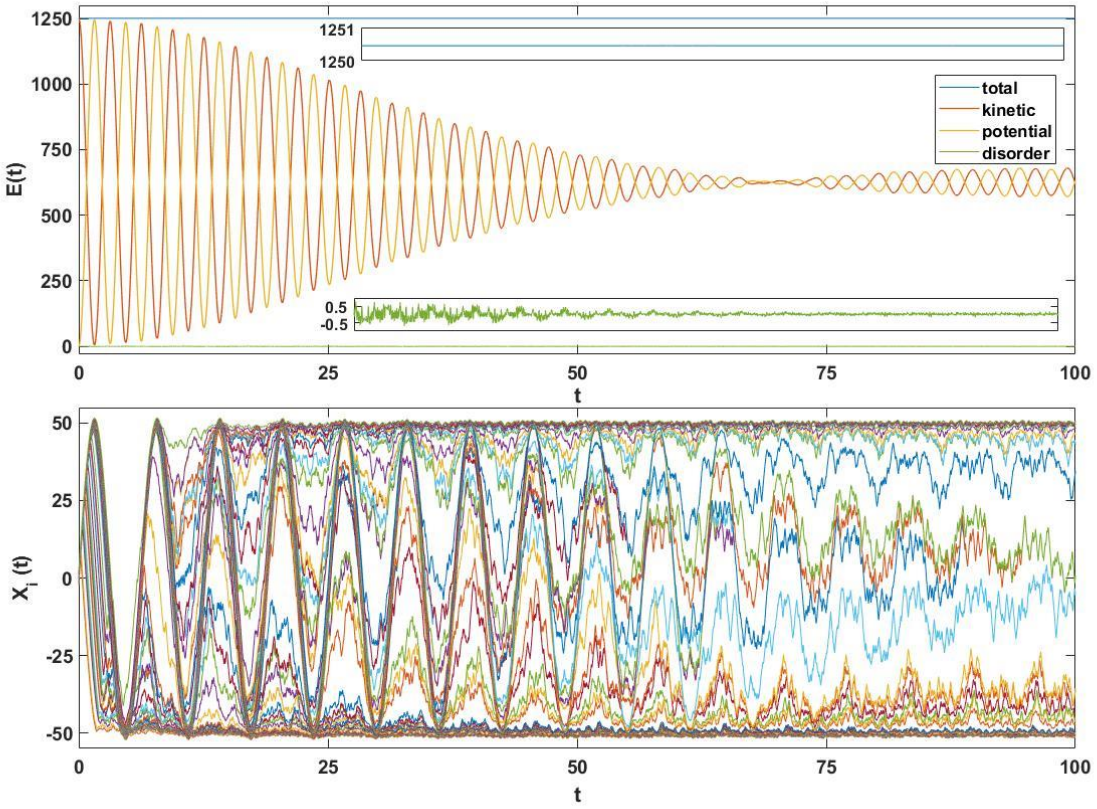

Fig. 2: *Top Panel: Time evolution of energies E(t): total (blue), kinetic (red), harmonic plus disorder potentials (yellow), only disorder (green); insets: zooms showing total (upper) and disorder (lower) energies. Bottom Panel: Bohmian trajectories $X_i(t)$, i=1…100. Atomic units are implicit.*