Posters

P:01 Using vacancy transport to unify memristor models

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This research demonstrates how simple closed form and numerical solutions for memristive characteristics can be generated from the transport equation.

The memristor has hidden in plain view for over two centuries [1]. Contemporary models continue to be built around V, Irelationships [2], [3]. An early attempt to inherently model nonlinearity without arbitrary window functions is by Lu [4]. A numerical model adds depth but is unable to generate an insightful abstraction [5]. This research demonstrates that vacancy transport can bind the various models, while bridging vacancy and circuit abstractions.

The Modeling Methodology

The transport partial differential equation (PDE) is proposed as the governing equation for the vacancy migration memristor. In its general form the transport PDE contains diffusion and generation-recombination (G-R) terms as in Table 1 equation (1). is the normalized vacancy concentration, $\vartheta$is evolution velocity in (x, t) , $\mathcal{D}$is diffusion constant and $\tau_{GR}$is the G-R coefficient. A symbolic solution of the form (2) is demonstrable when second order diffusion and G-R effects are ignored. The solution is inherently nonlinear, without window functions. (1) can be numerically solved with initial and boundary conditions generated from (2). Fig. 1 compares the symbolic and numeric solutions showing good correlation. Manipulating with methods from electrochemistry results in the closed form resistance formula (3). Variable Ø is the time-integral of voltage. Computing with (3) reveals the memristor as a two terminal device composed of two series complex resistors (4) that contain the phase information observed by Chua [6]. Numerically the imaginary parts in the two resistors cancel, leaving behind the ubiquitous (but in this case, improved) HP's dual variable resistor model (HP DVR) [3]. The larger of a, c in (4) takes on the positive sign, resulting in an always positive $R(\emptyset)$. Key result in Fig. 2(a) locates the vacancy accumulation boundary. Fig. 2(b) confirms vacancy conservation w.r.t. time. Fig. 2(c) shows current voltage I-V curve, the device progressively entering higher resistance. Fig. 2(d) shows that the model produces comparable symbolic (no diffusion or G-R) and numerical (diffusion and G-R) solution for resistance. Fig. 3 demonstrates reduced switching resistance range for increasing temperature. The memristor is ohmic in the ON region ($R \propto T$) and semiconducting in the OFF region ($R \propto T^{-1}$) [7]. This research introduces an improved nonlinear vacancy accumulation boundary similar to in [3], a new concept of symmetry boundary which is always a location 0 ≤ $x_s(t) \leq d$ [8], where is device length. The $x_s(t)$ information can be used to confine the model to compute exclusively between a low and high resistance. The symbolic (and equivalent numeric) modeling can compute the switching time (5), energy (6) and a nonlinear vacancy evolution velocity (Fig. 4). With u_t+ Du_{xx}= 0 we can compute vacancy evolution (un-programming) on the shelf. Fig. 5 is the solution stack generated from a single PDE.

This research uniquely unifies memristor modeling under the umbrella of vacancy transport.

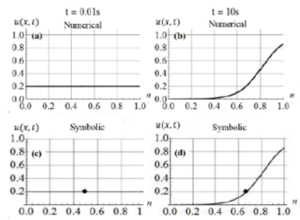

Fig. 1. (a), (b) show numerical results for vacancy evolution with $\mathcal{D} = \tau_{GR}^{-1} = 0$ while (c), (d) show matching results from using symbolic formula.

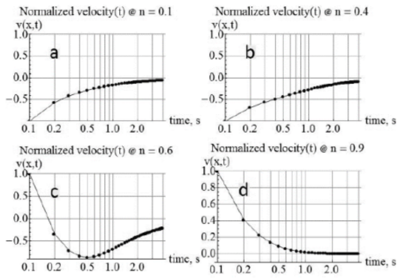

Fig. 4. Normalized $u(x,t)$ evolution velocity $\vartheta(x,t)$.

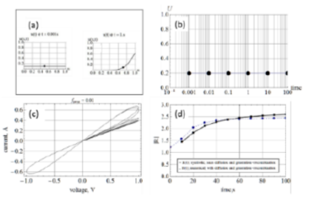

Fig. 2. Collage of results. (a) Evolution of $u(x,t)$. (b) Conservation of vacancies (c) Current-Voltage (I-V) curve and (d) symbolic, numeric overlay.

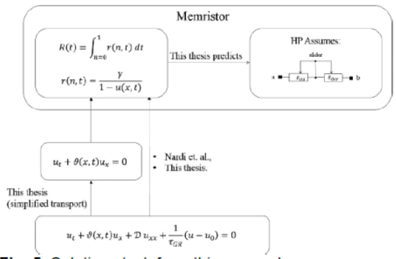

Fig. 5. Solution stack from this research.

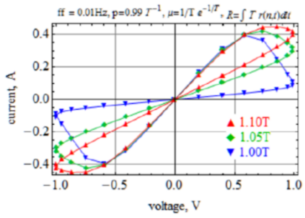

Fig. 3. Temperature dependence of I-V.

Table 1. Equations.

$u_t + \vartheta\, u_x + \mathcal{D}\, u_{xx} + \tau_{GR}^{-1}(u - u_0) = 0$	(1)
$u(x,t) = (1 + a\, e^{-g(x) f(x,t) h(t)})^{-1}$	(2)
$R(\emptyset) = \sum_{n=1}^{3} C_n\, f_n(\emptyset)$	(3)
$R(\emptyset) = (\pm a \mp i\, b) + (\mp c \pm i\, b)$	(4)
$\tau = f(\emptyset) \ln(\alpha)$	(5)
$E = \int_{t=0}^{\tau} \frac{V^2}{R(t)}\, dt$	(6)

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