The Effect of Size Scaling on the Magneto-transport Fluctuations in Ballistic Quantum Dots

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1. Introduction

Various effects are prevalent in the scaling of semiconductor devices. As the size of devices shrink, we find that there is a transition from classical to quantum phenomena. Similar in nature to the above, scaling also plays a vital role in magneto-transport fluctuations in ballistic dots as they are reduced in size. Experimental studies have revealed a high degree of periodicity in the conductance fluctuations, with only a few apparently harmonically related frequencies dominating the power spectra. Experimental data [1] appeared to suggest that the dominant frequency scales with $A^{1/2}$, where A is the area of the dot. On the other hand, a semiclassical analysis [2] of periodic orbits suggests that the scaling should be with area. Numerical simulations have been performed on nominally square quantum dots in order to determine the magneto-transport and corresponding wave-functions [3], but these results were inconclusive Here, we pursue a different approach, based upon the energy spectra of the dots. The results obtained from these calculations allows us to probe issues regarding the scaling of the dominant frequency of the fluctuations. From our numerical analysis we attempt to resolve this issue by simulating dots of many different sizes using a quantum mechanical approach.

The experimental studies, with which our numerical results are compared, have been performed on strongly confined, nominally square open quantum dots that were fabricated in GaAs/AlGaAs heterojunction material where the gates were patterned using electron beam lithography. The devices varied in size from 0.3 μm to 1.8 μm, where the dot size was determined from Aharonov-Bohm oscillations at high fields. The input and output leads were placed at the top corners of the dot, leading to a configuration that was highly effective in trapping electrons due to the effects of beam collimation. In particular, the storage time for electrons is two orders of magnitude larger than the ballistic transit time across the devices which is in the range of several hundred pico-seconds. If we consider that electrons trapped in the dot cavity cool to an effective temperature of 50 mK [4] along with duration of the storage time, the magnitude of the broadening of the discrete levels in the dot can be estimated at about 70 mK. This value is much smaller than the average level spacing for each of the dot sizes indicated above with the exception of the 1.8 μm dot. This indicates that the quantum mechanical nature of the transport in the dot becomes more resolved as the dot sizes are reduced in the experiment.

At low magnetic fields (≤0.3 T), the experimental magneto-resistance was found to be dominated by dense and reproducible fluctuations. Regular fluctuations are found to be an intrinsic feature of both experiment [5] and simulation [6] as found from studies of magneto-resistance plotted as a function of cavity size and gate voltage.

In the analysis of the data of the experiments as well as in earlier simulations, a Fourier analysis was performed on the conductance fluctuations to quantify the periodicities. In this paper, we use a different approach in trying to quantify the periodicities evident in the simulated conductance data. Specifically, we examine the conductance, G, as a function of *both* Fermi energy, E, and magnetic field, B, for several dot sizes corresponding to the experiments. As will become apparent, studying the three dimensional function $G(E,B)$ provides a very graphical and unambigious way to see first hand the dominant periodicities that occur in the conductance. Morever, the periodicities that are obtained agree well with the experiments.

2. Method of Calculation

In order to obtain a better understanding of the behavior of the magneto-conductance fluctuations within square quantum dots we have carried out simulations of the quantum transport through the dot for dot sizes of 0.15-0.8μm. In order to conduct quantum transport calculations we lay each dot out on a mesh as displayed in Fig. 1. The general situation is one in which ideal quantum wires, which extend outward to ±∞, are connected to the quantum dot. This quantum mechanical problem can be solved by using an iterative matrix method [7] applied to the discretized version of the Schrödinger equation, obtained by keeping terms up to first order in the approximation of the derivative:

$$(E_F - \mathbf{H}_j)\psi_j + \mathbf{H}_{j,j-1}\psi_{j-1} + \mathbf{H}_{j,j+1}\psi_{j+1} = 0 \qquad (1)$$

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where ψ_j is a M-dimensional vector containing the amplitudes of the jth slice. The problem is solved on a square lattice of lattice constant a with the wires extending M lattice sites across in the x direction and the region of interest being broken down into a series of slices along the y direction. In this equation, the $\mathbf{H}_j$ matrices represent Hamiltonians for individual slices and the matrices $\mathbf{H}_{j,j-1}$ and $\mathbf{H}_{j,j+1}$ give the inter-slice coupling. By approximating the derivative, the kinetic energy terms of Schrödinger's equation get mapped onto a tight-binding model with $t = -\hbar^2/2m^*a^2$ representing nearest neighbor hopping. The potential simply adds to the on site energies.

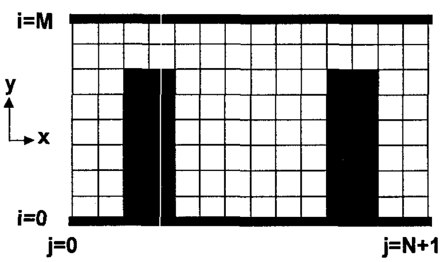

Fig. 1. The geometry of the quantum dot, and quantum point contacts, that are considered in this study. The grid represents the underlying mesh on which the calculations are performed, though in practice the mesh is much finer.

Equation (1) can be used to derive a transfer matrix which allows us to translate across the system and thus calculate the transmission coefficients which enter the Landauer-Büttiker formula to give the conductance. Transfer matrices however are notoriously unstable due to the exponentially growing and decaying contributions of evanescent modes. This difficulty can be overcome by performing some clever matrix manipulations and calculating the transmission by a iterative procedure rather than just multiplying transfer matrices together. The full details of this technique are given in ref. 7. This method in some ways is quite similar to the recursive Green's function techniques [8] that typically are used to solve these problems, and a comparison has shown good agreement between the two methods. The amplitudes of the wave functions at specific values of x and y can be found easily by backward substitution after the iteration is performed.

3. Conductance "Spectrum" and Scars

As mentioned earlier, previous work focussed on taking Fourier transforms of the conductance fluctuations resulting from the numerical simulations. The periodicity of the magnetoconductance was determined by identifying the dominant peaks of the power spectrum .

Unfortunately, the Fourier method can yield results that are difficult to interpret. In particular, peaks can split or merge depending on the details of the analysis. Here we overcome this problem by plotting conductance as a function of *both* magnetic field, B, and Fermi energy, E_F using the method described above to perform the calculations. Examination of such plots allows one to pick out periodicities that occur in the fluctuations directly and unambiguously. Fig. 2 (a) shows an example of a multi-dimensional plot for a 0.25 μm square dot. The conductance is indicated by the shading, with lighter shading corresponding to higher conductance. Note that the conductance is overlayed by resonance lines or *striations*. We have found that the positions of these resonance lines coincide closely with the spectrum of a *closed* dot of the same dimension [9-11]. While close, this correspondence is not exact, as opening the dot to the external environment broadens the energy levels. We have found that the leads play a crucial role in determining which closed dot states survive to yield conductance resonances. Importantly, the fact that we get such striated patterns is strong evidence that the dot spectrum is still being resolved in these open dots.

Many of the striations appear to be parallel to each other. The spacing of the parallel striations shown on the plot defines the periodicity of the fluctuations, which are essentially pinned by the underlying spectrum. Darker colored striations indicate conductance minimas while lighter ones indicate conductance maximas. The spacing of the striations should always be determined from adjacent minima or maxima. Many of these striations result in a wave function that is scarred. This scarring can be thought of as the quantum signature of trapped classical periodic orbits within the dot. An example of a scar corresponding to a diamond shaped orbit is shown in Fig. 2 (b). As the magnetic field or Fermi energy is varied to a position away from the striations the pattern of the wave function changes. Fig. 2 (c) shows a V-shaped pattern, which reflects the fact that the electrons are forced to enter the dot in a collimated and angled beam due to the quantization of the propagating modes in the lead [12]. It is this beam collimation that performs the spectral selection process noted above. Fig. 2 (d) shows another diamond scar, occurring along the same striation as Fig. 2 (b). Fig. 2 (e) is a diamond scar that falls on an adjacent parallel striation. Thus, particularly striations can be associated with specific scarred states and their associated orbits. The occurrence of parallel striations scarred by the same orbit is clear evidence that the associated periodicity that is obtained can be ascribed to that orbit. The periodicity of the diamond scar is the same as that of the fluctuations seen in experiment [5]. Hence, we use the periodicities of these striations to determine the magnetic frequencies for the dot.

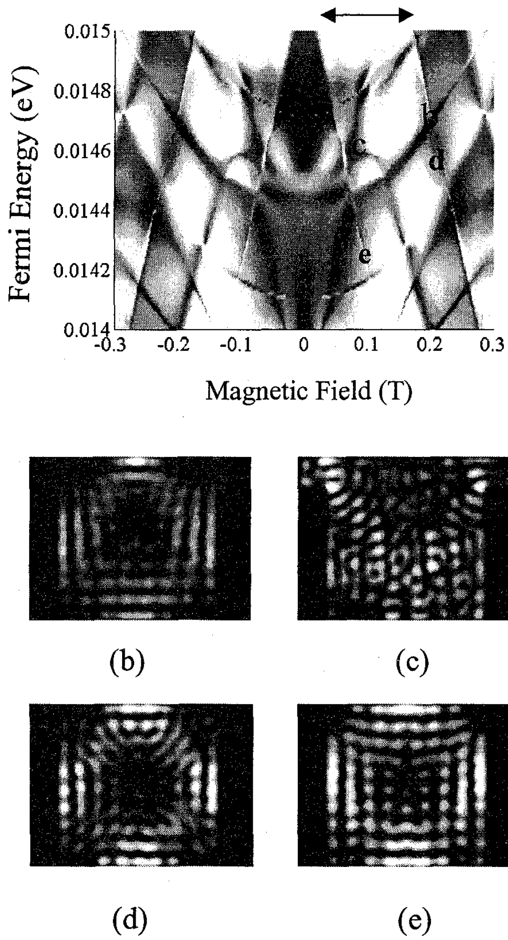

Fig. 2. (a) *G(E,B)* for a 0.25 μm square dot. (b) Wave function corresponding to point b in (a), E_F = 0.1470 eV and B=0.2029 T. (c) Wave function at point c in (a), E_F =0.1460 eV, B=0.0786 T. (d) Wave function corresponding to point d, E_F = 0.1453 eV, B = 0.2209 T. (e) Wave function corresponding to point e, E_F=0.1422 B=0.0993 T.

Fig. 3 (a) shows another example of a multi-dimensional plot for a 0.4 μm square dot in this case. Included in Fig. 3 are two notable wavefunctions which confirms the idea that wave function scarring is associated with these striations. Importantly, as this is a larger dot, much more complicated orbits can be resolved in the scarred wave functions. As one would expect, there are many more striations occurring for this dot than in the smaller dot used in Fig. 2. However, as with the smaller dot, striations can occur in parallel groups. In this case, the "double diamond" scar shown in Fig. 3 (c) appears to yield the dominant periodicity for this larger dot. The diamond scar also occurs in this dot, but is significantly weaker and harder to find. It should be noted that the diamond has a similar periodicity to the double diamond in this particular dot. The orbits reflected by figures 3 (b) and (c) can not be resolved in the smaller dot.

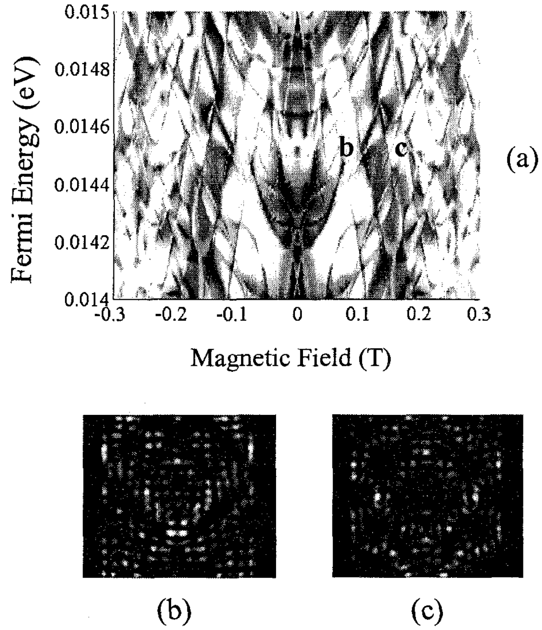

Fig. 3. (a) Conductance as a function of magnetic field and Fermi Energy for a 0.40 μm square dot. (b) Wave function corresponding to point b in (a), E_F =0.145 eV, B=0.074 T. (c) Wave function corresponding to point c in (a), E_F =0.145 eV, B=0.182 T.

Analysis of several dot sizes has allowed us to generate data to compare with the experimental results. These data points have been included in Fig. 4. The theoretical points have been determined by the spacings of the parallel striations associated with a particular orbit (e.g. note the arrow in Fig. 2). It is clear from the figure that the multi-dimensional analysis agrees quite well with the Fourier analysis of the dominant frequency obtained in the experiments. As Fig. 4 indicates, there is a transition in behavior that occurs as the dot edge length is varied. In particular, there is a notable transition which occurs at ~0.5 μm indicating that below ~0.5 μm, the periodic fluctuations scale with area, while above ~0.5 μm we find the scaling is actually with length. Analysis of the three dimensional conductance plots provides some evidence of what may be occurring within the dot. Our results clearly show that striations evolve and change location in magnetic field and energy as the dot size is increased and new structure emerges on the conductance plots. Since the periodicity is determined from the spacing between the striations it follows that any loss of striations changes the periodicity of the fluctuations, and lowers the magnetic frequency. Orbits less dominant or nonresolvable in the smaller dots eventually become more dominant as the dot size is increased.

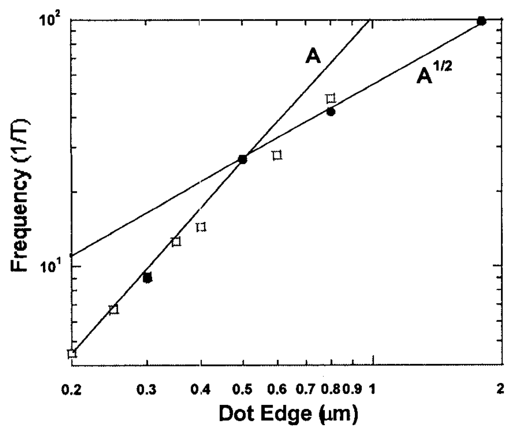

Fig. 4. The magnetic frequency as a function of the dot edge length for the numerical simulation data (open squares) obtained from the three-dimensional conductance plots mentioned in the text and the experimental data (solid circles). The numerical data was determined from the spacing of the striations on the conductance plots while the experimental data was found from the Fourier transforms of the magnetoconductance fluctuations in the square dot. The lines are a guide to the eye and identify different dependencies.

4. Conclusions

Through the use of an iterative stabilized mode matching technique we have been able to generate three-dimensional plots of conductance as a function of magnetic field and Fermi energy. Analyzing such plots has allowed us to determine the periodicities of the magneto-conductance fluctuations in square quantum dots. Extremely good agreement is found between the data obtained from experiment and that calculated in the numerical simulations. Both the experiment and simulation provide evidence that there is a transition in the scaling behavior of the periodicities as the dot size is varied. We believe that this is a result of other less dominant orbits in smaller dots eventually becoming the more dominant ones in larger dots due to the increase in complexity of the *G(E,B)* plots.

Acknowledgements

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