MAGNETIC QUASI-BOUND-STATE INDUCED RESONANT COUPLING OF EDGE STATES

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Abstract

We numerically examine the ballistic transport properties of an electron channel with a single scatterer, an antidot, when a perpendicular magnetic field is present. Formation of magnetic quasi-bound-states (MQBS) is observed in such a structure. The MQBS's couple magnetic edge states, resulting in resonances. In the multiple mode regime, coupling can occur between opposite edge states, resulting in resonant reflection, or anti-resonance. An edge state can also tunnel through the scattering region via an MQBS, resulting in resonant transmission. These resonances are closely related to those observed in quasi-one-dimensional systems such as T structures where such resonances are associated with transmission poles in the complex energy plane.

I. INTRODUCTION

For a quantum channel with an impurity in a perpendicular magnetic field, there exist three types of electronic states: (1) Circulating Landau states in the bulk region of the channel, (2) localized states corresponding to the circulating orbits around the impurity, and (3) extended states corresponding to classical skipping orbits near the channel walls, illustrated in Figure 1. The first type is highly degenerate therefore does not carry net current; the second type forms magnetic bound-states, which do not carry net current either; the third type forms magnetic edge states, which move in opposite directions on opposite walls, carrying net current. For a wide channel in high magnetic field, the impurity potential does not couple edge states on opposite walls, therefore back-scattering is suppressed. This implies unity transmission probabilities for edge states, leading to the integer quantization of Hall resistance [1]. For a narrower channel where the extension of the circulating orbits around the impurity overlaps with the edge states, magnetic quasi-bound-states (MQBS) form. Through the interference of MQBS's, an edge state on the upper wall can be back-scattered to its counterpart on the lower wall, resulting in resonant reflection; or an edge state on the left side of the impurity can tunnel to the same edge state on the right side of the impurity on the same wall, resulting in resonant transmission. This resonance phenomenon was suggested by Jain and Kivelson in a semiclassical calculation [2]. In this paper we present a quantum mechanical calculation of an electron channel with an antidot by using the Quantum Transmitting Boundary Method (QTBM) in which magnetic field is taken into account in the whole device, including the lead regions [3]. The suggested resonances are confirmed in our results.

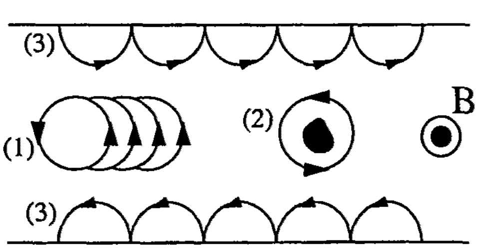

Figure 1. Electronic states in a quantum channel with an impurity. Circulating orbits around the impurity form magnetic bound-states or magnetic quasi-bound states; skipping orbits on the channel walls form magnetic edge states.

II. MODEL AND METHODS

Figure 2 shows the schematic geometry. The channel width is d, the radius of the antidot r. Device domain is marked by the dashed lines with an extension a in $\hat{x}$-direction. We present the results for the particular case where $r/d=1/20$. By including enough evanescent modes in the QTMB calculation, a can be chosen arbitrarily provided that the device region encloses the scatterer (antidot). We adopt the single-band, effective mass model with $m/m^*=0.067$, appropriate for GaAs. Hard wall potentials are assumed to define the channel edges and the antidot area and zero potentials assumed elsewhere in the channel. We choose the vector potential in the Landau gauge, $\vec{A} = -By\hat{x}$. The two-dimensional Schrödinger equation becomes

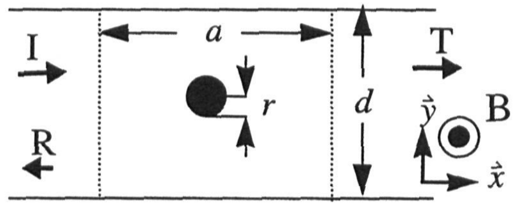

Figure 2. Schematic geometry of a quantum channel with one antidot as a single scatterer in a perpendicular magnetic field.

$$\left(\frac{-\hbar^2}{2m^*}\nabla^2 + \frac{i|e|\hbar By}{m^*}\frac{\partial}{\partial x} + \left(\frac{|e|^2B^2y^2}{2m^*}\right) + V_0(x,y)\right)\psi(x,y) = E\psi(x,y)\,, \tag{1}$$

where $V_0(x,y)$ is the potential in the device region. We employ the QTBM to solve Equation (1) for the scattering state. Boundary conditions are implemented in the QTBM by expanding the scattering state in a lead region as a superposition of the local transverse (including both traveling and evanescent) modes. Transverse modes $\{\varphi_n(y)\}$ are obtained by using the form $\psi_i(x,y) = exp(ik_nx)\varphi_n(y)$ for lead region i. The original Schrödinger equation becomes a quadratic eigenvalue problem for wavevector k_n at given energy E

$$\left(-\frac{\partial^2}{\partial y^2} + \left(\frac{eBy}{\hbar} + k_n\right)^2 + \frac{2m^*}{\hbar^2}V_i(y)\right)\varphi_n(y) = \frac{2m^*}{\hbar^2}E\varphi_n(y)\,. \tag{2}$$

From the full wavefunction solution, complex energy-dependent transmission and reflection amplitudes for each transverse mode are obtained. Then we use the two-terminal Landauer formula to obtain the conductance in the linear response regime, $G = (2e^2/h)Tr(tt^{\dagger})$. We also compute the particle current density in the device region from the following definition

$$\vec{j} = \frac{i\hbar}{m^*}(\psi\nabla\psi^* - \psi^*\nabla\psi) + \frac{|e|\vec{A}}{m^*}|\psi|^2. \tag{3}$$

III. RESULTS AND DISCUSSIONS

In Figure 3 we plot the conductance as a function of magnetic field and incident energy. The strength of the field is measured by the parameter $\beta = eBd^2/\hbar = d^2/l_H^2$ where l_H is the magnetic length. Energy is expressed in units of the first bulk Landau level $E_L(\beta) = \hbar\omega_c/2 = \hbar eB/2m^*$. Notice that the energy units $E_L(\beta)$ are different for different fields. In the inset we plot the individual transmission coefficients T_1 and T_2 at β=40 when the incident electrons are in the first edge state and second edge state respectively.

Should there be no scatterer (antidot), the conductance would be a series of smooth platforms with its height corresponding to the number of existing edge channels. In the presence of the antidot, however, scattering of edge states takes place and more structure develops in the conductance. Particularly, the spikes, as in a one-dimensional double barrier device, indicate certain resonant processes. They are evident at high fields in Figure 3, with dips indicating resonant reflection and peaks resonant transmission. When the field is lowered, both the dips and the peaks are broadened and smoothed and they eventually disappear in very low fields. We now examine the wavefunctions of the electron scattering states for such resonant reflection state and resonant transmission state.

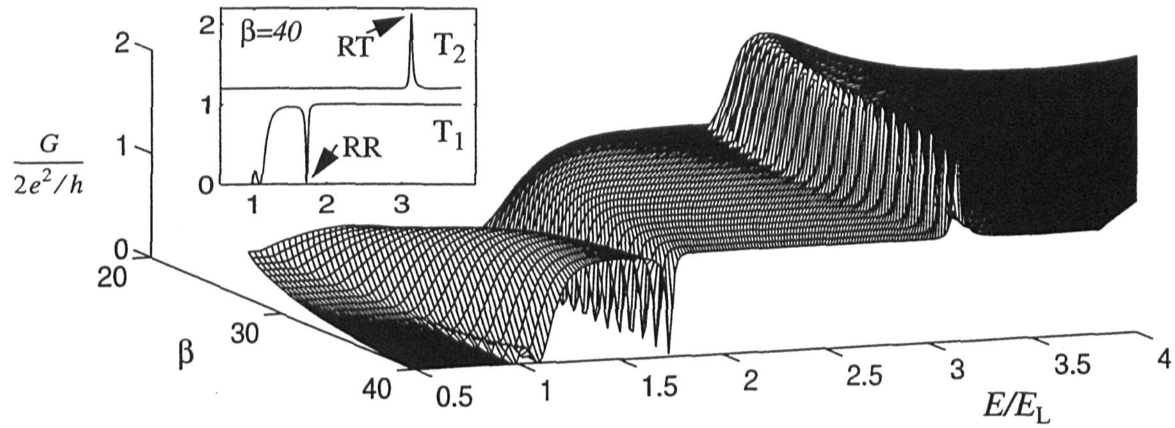

Figure 3. Conductance verses Energy E and magnetic field β. E is expressed in units of the first bulk Landau level $E_L(\beta)=\hbar\omega_c(\beta)/2$. Inset: modal transmission coefficients for the first edge state (T_1) and second edge states (T_2) at magnetic field $\beta=40$. RR is a resonantly reflected state, RT a resonantly transmitted state.

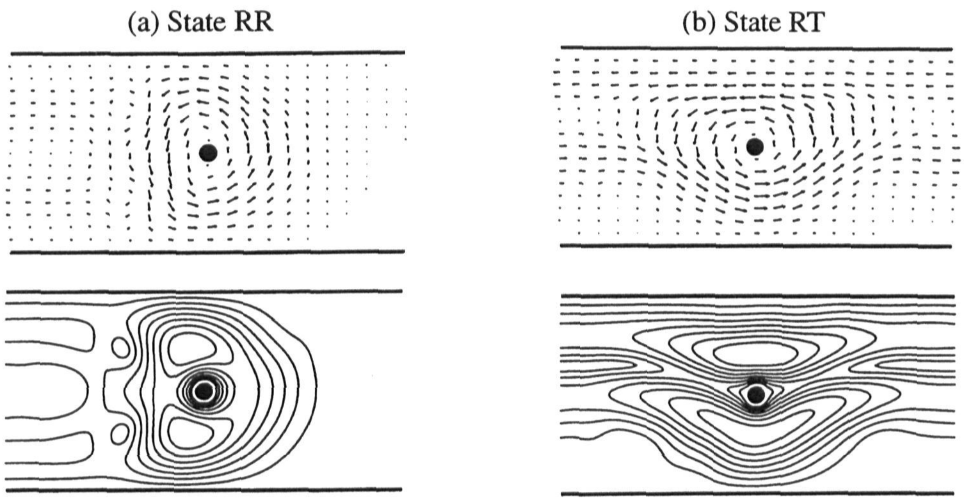

Figure 4. Particle current density (top) and probability density (bottom) distribution. (a) Resonant reflection state RR. (b) Resonant transmission state RT.

In Figures 4 we plot the distribution of the particle current density (vector field) and the probability density (contour lines) in a domain of $a=2d$ for states RR and RT indicated in the inset of Figure 3. In both cases strong circulating orbits are observed around the antidot, forming magnetic quasi-bound states. For state RR, the incident wave is in the first edge mode on the left upper wall; through the MQBS, it scatters to the first edge mode on the left lower wall, resulting in resonant reflection. For state RT, the incident wave is in the second edge mode on the left upper wall; through the MQBS, it tunnels to the second edge mode on the right upper wall, giving rise to resonant transmission. For a 1D double barrier (or T stub) structure, resonant tunneling (or reflection) occurs when the incident energy coincides with the energy levels of the quasi-bound states formed in the potential well (or in the stub); transmission probability on the real energy axis can be deduced from the positions of reflection zeros (or transmission zeros for T stub) and transmission poles on the complex energy plane, both of which can be obtained by using an eigenvalue technique[4]. For our quasi 1D channel in a magnetic field, resonances are likewise induced by the quasi-bound states in the system, only now they are of magnetic in origin and formed around the antidot. The physical association between resonance and magnetic quasi-bound state is nevertheless evidently shown in our numerical calculation. So, when the magnetic field is decreased, the circulating orbit is held less tightly to the antidot and magnetic quasi-bound states eventually dissolves into the extended states. Interference between edge states due to the scattering potential of the antidot still occurs but resonances disappear.

We further introduce an additional stripe of potential barrier V_0 to the channel and the conductance results at $\beta=40$ are shown in Figure 5 for $V_0/E_L=0, 1, 3, 5$ and $r/d=1/20$ and $w/d=1/20$. In a wave packet calculation, a similar (but wider) structure was used by Müller to illustrate the lack of destructive interference of edge states where the potential barrier was mainly considered as to force the incident wave to split into a tunneling part directly through the potential stripe and a scattered part via the antidot [5]. Our geometry shows clear destructive interference manifested by the dips and peaks in conductance curves in Figure 5. However, the resonances, both reflection and transmission, evident at V_0=0.0, are weakened for non-zero V_0's. So, by introducing more overlaps between the circulating orbits and the edge states, one effect of the potential stripe is to lessen even eliminate the formation of MQBS's, hence broaden even lift the resonances.

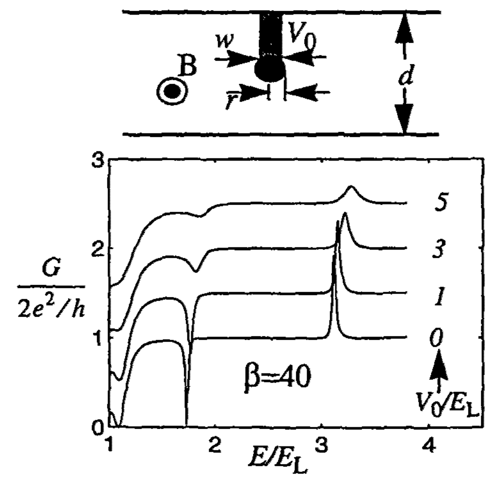

Figure 5. Conductance of a quantum channel with one antidot and a strip of potential barrier V_0. for $V_0/E_L=0, 1, 3$ and 5 at $\beta=40$.

IV. SUMMARY

We have calculated the magneto transport properties of a quantum channel with a single antidot. Resonant reflection and resonant transmission are observed at high magnetic fields. They are induced by the tunneling processes of the edge states, via the magnetic quasi-bound states formed around the antidot. The resonances are broadened in lower magnetic fields because the MQBS's are less localized. The smoothing of resonances is also observed when an additional potential barrier is introduced in the channel which also tend to hinder the formation of MQBS's.

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