CONFINED ACOUSTIC PHONON CONTROLLED RELAXATION TIMES IN FREE STANDING QUANTUM WELLS

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Abstract

The scattering rates for electron – confined acoustic phonon interactions in free-standing quantum wells are calculated numerically. We have considered the relaxation times in the test particle approximation as well as in the approximation corresponding to the kinetic equation solution through the polar functions expansion. The quantization of acoustic phonons in free-standing quantum wells results in peculiarities in transport coefficients.

I. INTRODUCTION

Free-standing quantum structures attract considerable attention because they are promising for optoelectronic and electronic applications, as sensitive sensors, and for probing the local properties of solids. Furthermore, the free-standing structures are very interesting physical objects which display new physical phenomena and they are challenging objects for nanotechnology. The main feature of the acoustic phonon subsystems in free-standing structures is the quantization of the acoustic phonon wavevectors in the direction of the confinement. This quantization is responsible for the peculiarities of the acoustic phonon interactions with electrons and photons displayed as a set of peaks in the differential conductances [1] and luminescence spectra [2] of quantum microstructures. In this paper we concentrate our attention on the electron - confined acoustic phonon interactions in free-standing quantum wells (FSQWs) and on the peculiarities of the transport coefficients due to the acoustic phonon quantization. We have developed a model of the electron scattering by confined acoustic phonons interacting through the deformation potential. This model is used in collision integral of the kinetic equation. We solved the kinetic equation both in the test particle approximation and reducing it to the Fredholm equation of the second kind.

II. ELECTRON – CONFINED ACOUSTIC PHONON SCATTERING

We will consider FSQW of width a. The electron wavefunctions will be taken in the approximation of the infinitely deep quantum well. The electron states are characterized by the in-plane wavevector $\mathbf{k}_{\parallel}$ and the subband number n. The acoustic phonon eigenmodes in FSQW and their dispersion relations were obtained in [3, 4, 5]. The acoustic phonons are characterized by the in-plane wave vector $\mathbf{q}_{\parallel}$, the mode number m, and the symmetry α.

In accordance with the Fermi golden rule the probability density for the electron transition $(\mathbf{k}_{\parallel}, n) \rightarrow (\mathbf{k}'_{\parallel}, n')$ due to the confined phonon absorption (upper sign) or emission (lower sign) is given by the formula

$$W^{\left\{ ab \atop em \right\}}_{\mathbf{k}_{\parallel},n \rightarrow \mathbf{k}'_{\parallel},n'} = \frac{\pi\, E_a^2 \,(n^{\alpha}_{\mathbf{q}_{\parallel},m} + \frac{1}{2} \mp \frac{1}{2}) \mid F_{\alpha,m} \mid^2 \left(q^2_{t,m} - {q_x}^2\right)^2 \left(q^2_{l,m} + {q_x}^2\right)^2}{\mathcal{A}\, \rho\, \omega^{(\alpha)}_m(\mathbf{q}_{\parallel})} \times \tag{1}$$

$$\times\; tsc^2_{\alpha}(\frac{a\, q_{t,m}}{2})\, \mathcal{G}(n', n, \alpha, q_{l,m})\, \delta_{\mathbf{k}_{\parallel} \pm \mathbf{q}_{\parallel}, \mathbf{k}'_{\parallel}}\, \delta(\varepsilon \pm \hbar\, \omega^{(\alpha)}_m(\mathbf{q}_{\parallel}) - \varepsilon')\,,$$

where we use the same notations as in Ref. [5, 6], the function $tsc_{\alpha} = \sin$, if $\alpha = \mathit{dilatational}$ and $tsc_{\alpha} = \cos$, if $\alpha = \mathit{flexural}$, $\mathcal{G}(n', n, \alpha, q)$ is the overlap integral.

To analyse the electron transport properties we will need scattering rates in the following form

$$\tau_G^{-1} = \sum_{n', \mathbf{k}'_{\|}, \alpha, m, \mathbf{q}_{\|}, \beta} W^{\beta}_{\mathbf{k}_{\|}, n \to \mathbf{k}'_{\|}, n'} \, G \,, \tag{2}$$

where β is either *absorption* or *emission*, G is some given function which may depend on all variables over which we take sum. We will also use $(\tau_G^{ab})^{-1}$ and $(\tau_G^{em})^{-1}$ which are defined in a similar way with the only distinction that we sum up either only absorption terms or only emission terms. There is an obvious relation between them: $\tau_G^{-1} = (\tau_G^{ab})^{-1} + (\tau_G^{em})^{-1}$. If we employ the formulae for transition probabilities (1) we may obtain the following relations for scattering rates

$$\left(\tau_G^{\left\{ab \atop em\right\}}\right)^{-1} = \frac{E_a^2 \, m}{2\pi \hbar^2 \rho k_{\|}} \sum_{n', \alpha, m} \sum_i \int_0^{\infty} dq_{\|} \, \mathcal{F}^{\left\{ab \atop em\right\}}(n', n, \alpha, m, q_{\|}) \, G \, \frac{1}{|\sin \Psi_i|} \,, \tag{3}$$

where

$$\mathcal{F}^{\left\{ab \atop em\right\}} = \frac{(n^{\alpha}_{\mathbf{q}_{\|}, m} + \frac{1}{2} \mp \frac{1}{2}) \, | \, F_{\alpha, m} \, |^2 \left(q_{t,m}^2 - {q_x}^2\right)^2 \left(q_{l,m}^2 + {q_x}^2\right)^2}{\omega_m^{(\alpha)}(\mathbf{q}_{\|})} \, tsc_{\alpha}^2\left(\frac{a \, q_{t,m}}{2}\right) \mathcal{G}(n', n, \alpha, q_{l,m}) \,,$$

and angles $\Psi_i \in [0, \pi]$ are solutions of the transcendental equation

$$\cos \Psi = \frac{m \, \omega_m^{(\alpha)}(q_{\|})}{\hbar \, k_{\|} \, q_{\|}} \pm \left(\frac{m \, (\varepsilon_n - \varepsilon_{n'})}{\hbar^2 \, k_{\|} \, q_{\|}} - \frac{q_{\|}}{2 \, k_{\|}} \right) .$$

III. RELAXATION TIMES IN THE TEST PARTICLE APPROXIMATION

In the test particle approximation, the scattering rate, τ, the momentum relaxation rate, τ_p, and the energy relaxation rate, τ_ε are given by the formulae

$$\tau^{-1} = \sum_{\mathbf{p}'} W_{\mathbf{p} \to \mathbf{p}'} \frac{1 - f^0_{\mathbf{p}'}}{1 - f^0_{\mathbf{p}}} \,, \tag{4}$$

$$\tau_p(\mathbf{p})^{-1} = \sum_{\mathbf{p}'} W_{\mathbf{p} \to \mathbf{p}'} \left[1 - \frac{p' \cos \psi}{p} \right] \frac{1 - f^0_{\mathbf{p}'}}{1 - f^0_{\mathbf{p}}} \,, \tag{5}$$

$$\tau_\varepsilon(\mathbf{p})^{-1} = \sum_{\mathbf{p}'} W_{\mathbf{p} \to \mathbf{p}'} \left[1 - \frac{\varepsilon'}{\varepsilon} \right] \frac{1 - f^0_{\mathbf{p}'}}{1 - f^0_{\mathbf{p}}} \,, \tag{6}$$

where $f^0_{\mathbf{p}}$ is the Fermi distribution function, ψ is the angle between $\mathbf{p}$ and $\mathbf{p}'$.

We have computed integrals in (4), (5), and (6) for the electron scattering by the dilatational phonons in the lowest electron subband numerically and obtained the scattering rates τ^{-1}, τ_p^{-1}, and τ_ε^{-1} as functions of energy. The calculations were made for $GaAs$ QW of width $a = 100\mathring{A}$, for temperatures $T = 300K$, $T = 77K$, and $T = 4.2K$, for both degenerate and nondegenerate electron gas. The degenerate electron gas was characterized by the Fermi energy $\mu = 50\,meV$ and corresponding to it the electron concentration $n_s = 1.4 \times 10^{12}\,cm^{-2}$. The most interesting dependences are depicted in the Fig. 1 through 4. Fig. 1 and 2 correspond to the case of nondegenerate electron gas at temperatures $T = 300K$ and $T = 4.2K$ accordingly and Fig. 3 and 4 correspond to the case of degenerate electron gas at temperatures $T = 77K$ and $T = 4.2K$

accordingly. The solid lines in the Fig. 1 through 4 correspond to the acoustic phonon emission and the dotted lines correspond to the acoustic phonon absorption. It should be noted that the relaxation times τ and τ_p are very similar, therefore we will provide the graphs of only τ_p. The quantities τ_ε^{-1} for phonon absorption are obviously negative due to the factor $[1-\varepsilon'/\varepsilon]$, however we use the same axes as for the energy relaxation rate corresponding to the phonon emission and plot them as positive functions.

IV. THE KINETIC RELAXATION TIME

The kinetic equation for the electron distribution function, f, may be solved in the case of small deviation from equilibrium. In this case $f = f^0_{\mathbf{p}} + \mathrm{f}_1\, \mathbf{p}/p$, where the nonequilibrium part of the distribution function may be represented in the form $\mathrm{f}_{1\mathbf{p}} = -\tau_1(p)\,\mathbf{F}\,\frac{\partial f^0_{\mathbf{p}}}{\partial p}$. The momentum relaxation time, τ_1 satisfies the Fredholm equation of the second kind

$$\tau_1(p) = \tau(p) + \tau(p) \sum_{\mathbf{p}'} W_{\mathbf{p}\to\mathbf{p}'} \frac{p' \cos\psi}{p}\, \tau_1(p')\, \frac{1-f^0_{\mathbf{p}'}}{1-f^0_{\mathbf{p}}}\,. \tag{7}$$

We solved Eq. (7) by iterations. The electron conductivity may be expressed through τ_1 in the following way

$$\sigma = \frac{e^2}{2\pi\hbar^2 m^2 T} \int_0^\infty dp\, p^3\, \tau_1(p)\, f^0_{\mathbf{p}}\,(1-f^0_{\mathbf{p}})\,.$$

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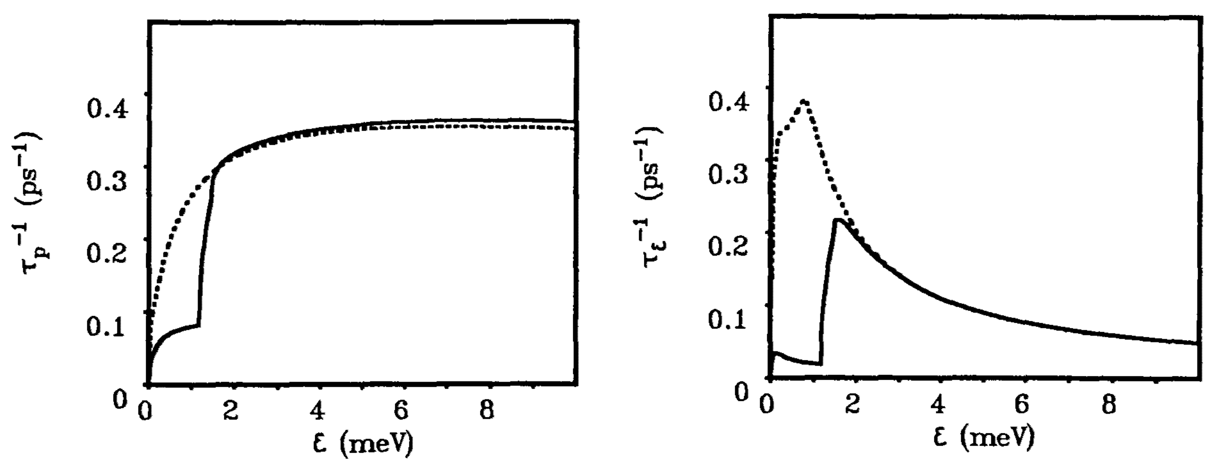

Figure 1. The momentum relaxation rate, τ_p^{-1}, (left) and the energy relaxation rate, τ_ε^{-1}, (right) in *GaAs* FSQW of width $a = 100\AA$. Nondegenerate case, $T = 300K$, solid line corresponds to the phonon emission, dotted line corresponds to the phonon absorption.

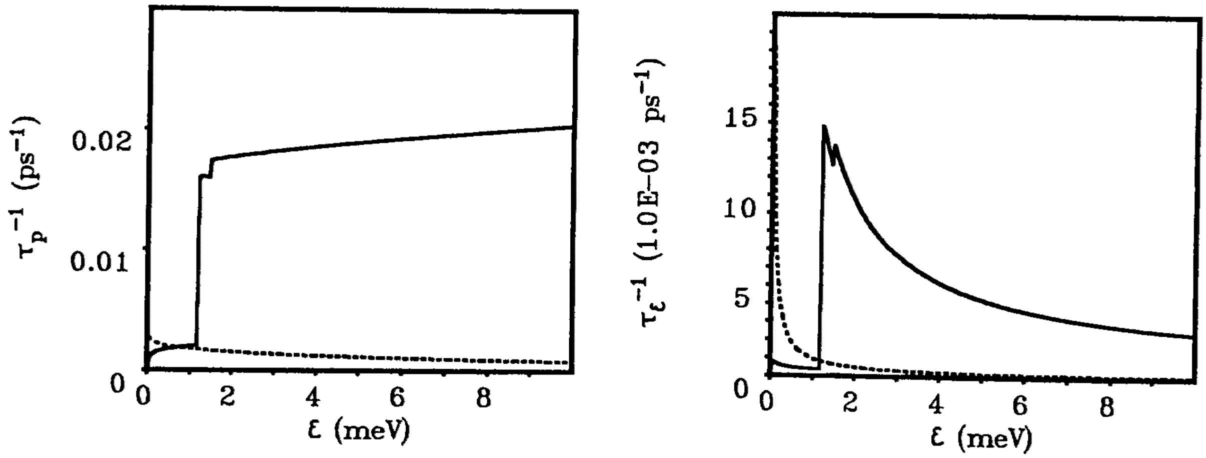

Figure 2. The momentum relaxation rate, τ_p^{-1}, (left) and the energy relaxation rate, τ_ε^{-1}, (right) in *GaAs* FSQW of width $a = 100\AA$. Nondegenerate case, $T = 4.2K$, solid line corresponds to the phonon emission, dotted line corresponds to the phonon absorption.

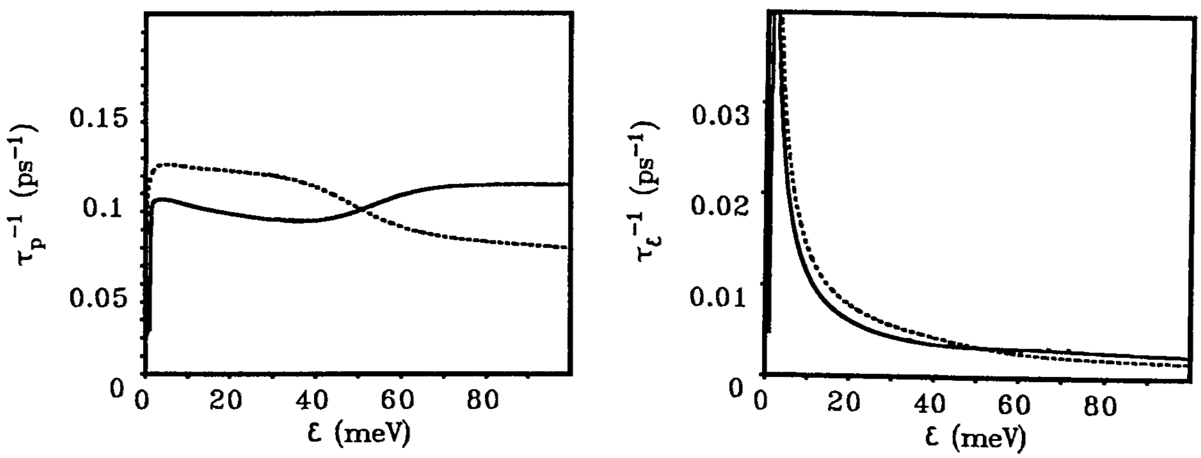

Figure 3. The momentum relaxation rate, τ_p^{-1}, (left) and the energy relaxation rate, τ_ε^{-1}, (right) in *GaAs* FSQW of width $a = 100\AA$. Degenerate case, $T = 77K$, solid line corresponds to the phonon emission, dotted line corresponds to the phonon absorption.

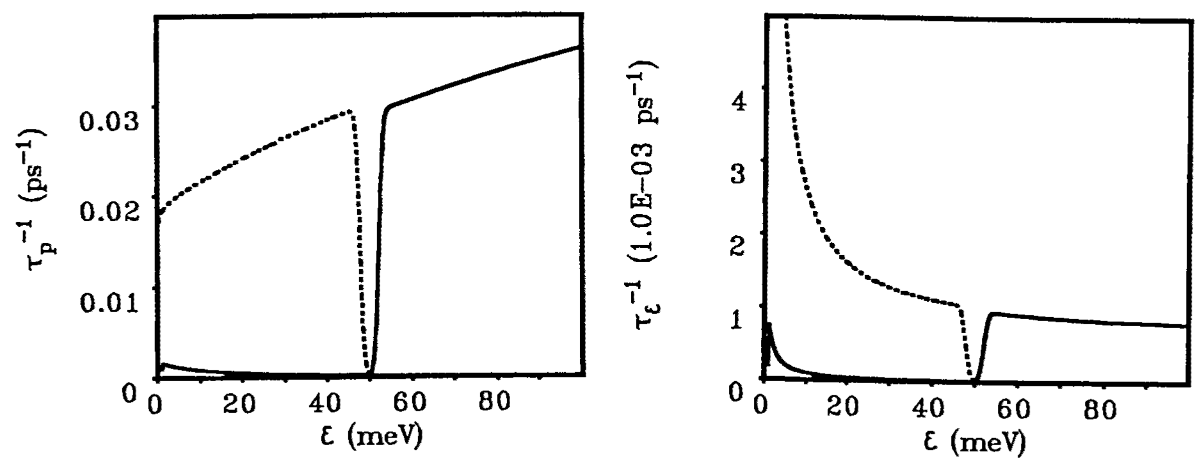

Figure 4. The momentum relaxation rate, τ_p^{-1}, (left) and the energy relaxation rate, τ_ε^{-1}, (right) in *GaAs* FSQW of width $a = 100\AA$. Degenerate case, $T = 4.2K$, solid line corresponds to the phonon emission, dotted line corresponds to the phonon absorption.