Tunneling and its Inclusion in Analytical Models for Abrupt HBTs

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Abstract
In an abrupt AlGaAs/GaAs HBT, transport in the Conduction Band Spike (CBS) can be the mechanism which limits the overall transport current within the HBT. In this paper closed-form analytic models are presented that describe the transport of carriers in the CBS. These models retain their connection to the physical attributes of the abrupt HBT, yet are simple enough to use in simulators such as SPICE.

I. INTRODUCTION

In an HBT with an abrupt emitter-base heterojunction, and a doping concentration in the narrow-bandgap base that is much larger than that in the wide-bandgap emitter, the Conduction Band Spike (CBS) is as shown in Fig. 1. As has been reported [1]-[5], this CBS plays a vital role in current transport within HBTs and can, in certain cases, completely determine the collector current [5]. Therefore, one must accurately model current transport in the CBS in order to accurately predict the performance of abrupt HBTs.

For the structure depicted in Fig.1, with base doping concentration around 10^{19} cm^{-3} and emitter doping around 10^{17} cm^{-3}, the relevant width of the CBS, as regards tunneling, is about 100 Å. As has been demonstrated by the aforementioned authors, based largely upon the work of Stratton, Padovani, and Christov [6]-[8], an accurate account of tunneling in abrupt HBTs is essential. The basic limitation of the published works regarding the modelling of CBS transport, is that the models can in general only be solved by appealing to numerical techniques; this hides the rich interplay that exists between the physical attributes such as doping concentration, temperature, effective mass, electron affinity, bias conditions, and the final transport model for the CBS. The work to be presented deals with the account of said tunneling in order to arrive at workable, analytic models for current transport across the CBS in abrupt HBTs.

II. THEORY OF CURRENT TRANSPORT IN ABRUPT HBTS WITH TUNNELING

Assuming that transport of electrons through the space charge region is not a limiting factor, then transport in the CBS is determined by both thermionic emission and tunneling. In such a case we find ([5], (32)) that the overall transport current J_C (which in general is equal to the collector current) within the HBT is given by:

$$J_C = q\gamma\upsilon\frac{n_{i,p}^2}{N_A}\exp\left[\frac{-\Delta E_{n0}}{kT}\right]\exp\left[q\frac{N_{rat}V_{BE}}{kT}\right] \tag{1}$$

where

$$N_{rat} = \frac{N_A}{N_A+N_D} \qquad \Delta E_{n0} = \Delta E_c - q\frac{N_D}{N_A}N_{rat}V_{bi} \qquad V_{bi} = \frac{kT}{q}\log\left(\frac{N_AN_D}{n_{i,p}^2}\right)+\Delta E_c$$

and using the notation and results of [1],

$$\gamma = 1+\exp\left[\frac{E_c(0^-)}{kT}\right]\frac{1}{kT}\int_{\max\,[E_c(x_p),\,0]}^{E_c(0^-)} \mathrm{D}\left(\frac{E_x}{E_c(0^-)}\right)\exp\left[\frac{-E_x}{kT}\right]\mathrm{d}E_x \qquad \upsilon = \left(\frac{kT}{2\pi m_n^*}\right)^{1/2} \tag{2}$$

with $N_{A(D)}$ being the base(emitter) doping concentration, $n_{i,p}$ the intrinsic carrier concentration in the base, ΔE_c the conduction band discontinuity, V_{bi} the built-in potential, V_{BE} the applied base-emitter potential, and υ the effective electron velocity that results from integrating over all particle velocities with some component parallel to the direction of the transport charge flow.

γ is the tunneling factor [1] and (2) is valid as long as the energies E_x over which there is significant contribution to the tunneling current are well above the electron quasi-Fermi energy. Furthermore, due to the limits of integration for γ, it is assumed that the transmission coefficient D(U) is unity above the CBS, i.e., the WKB approximation for the calculation of D(U) is asserted. Using the depletion approximation for an abrupt metallurgical junction, and assuming a coincident metallurgical/hetero-junction interface, D(U) is [1], [7]:

$$\mathrm{D}(U) = \exp\left[\frac{2}{q\hbar}\sqrt{\frac{m_n^* \varepsilon_n}{N_D}}E_c(0^-)\left(\log\left(\frac{\sqrt{1-U}+1}{\sqrt{U}}\right)U - \sqrt{1-U}\right)\right]. \quad (3)$$

In (3), U is the normalised energy and is given by $U=E_x/E_c(0^-)$, where $E_c(0^-)$ is the height of the CBS and is given by $E_c(0^-)=qN_{rat}(V_{bi}-V_{BE})$, i.e., $U=1$ at the top of the CBS. In order to gain a familiarity with (2) and (3), Fig. 2 plots the normalised emission flux density (given by the integrand in (2)) that emerges to the right of the CBS for an abrupt HBT with the following material parameters: N_D: $5\mathrm{x}10^{17}\,\mathrm{cm}^{-3}$; N_A: $1\mathrm{x}10^{19}\,\mathrm{cm}^{-3}$; ε_n: $12.2\varepsilon_0$; 30% Al in the emitter; ΔE_c: 0.24 eV; $n_{i,p}$: $2.25\mathrm{x}10^6\,\mathrm{cm}^{-3}$; $\rightarrow \Delta E_i$: 77.3 meV; V_{bi}: 1.671 V; m_n^*: $0.091m_0$. Unexpectedly, the normalised energy U for the peak emission flux density is not a function of applied bias. After some manipulation of the integrand in (2) (using (3)), it is found that the energy for peak emission U_{max}, and the normalised peak emission flux density F_{max} are:

$$F_{max} = \exp\left[-\frac{qN_{rat}(V_{bi}-V_{BE})}{kT}\frac{\tanh(U_p)}{U_p}\right] \qquad U_{max} = \cosh^{-2}(U_p) \qquad U_p = \frac{q\hbar}{2kT}\sqrt{\frac{N_D}{m_n^*\varepsilon_n}}.$$

The fact that U_{max} is independent of the applied potential is interesting in that, relative to the top of the CBS, the emitted electron flux density is always centred at the same place. Discussion of this result will follow in Section III.

Now, given that U_{max} is independent of applied potential, that F_{max} has an exponential characteristic, and the emission flux density has a highly symmetric shape (Fig. 2), there promises to be a potentially simple analytic result for evaluating γ in (2). Through a series of transformations the normalised emission flux can be written as:

$$\int \mathrm{D}(U)e^{\frac{-UE_c(0^-)}{kT}}dU = \int \frac{1}{r}\frac{dy}{dr}e^{C_1 y}dr \qquad C_1 = \frac{E_c(0^-)}{U_p kT} \qquad \begin{array}{l} y = r\cosh^{-2}(U_p+r) - \tanh(U_p+r) \\ U = \cosh^{-2}(U_p+r) \end{array} \quad (4)$$

Equation (4) provides for the exact solution to the tunneling current. If the transform function $y(r)$ were invertible so that $r(y)$ could be determined, then (4) would yield the desired solution in the y domain. However, $r(y)$ cannot be determined analytically in an exact form, but does yield to an approximate form, e.g., the second-order expansion given in (5). Using this approximate $y(r)$ to solve the integral in (4), with limits of $\pm\infty$ (which implies that most of the emission flux should be contained within the limits specified in (2)), then γ is given by:

$$\gamma \approx 1 + \sqrt{\frac{4\pi\sinh(U_p)U_pE_c(0^-)}{\cosh^3(U_p)kT}}\,e^{\frac{E_c(0^-)}{kT}\left(1-\frac{\tanh(U_p)}{U_p}\right)} \quad \text{where} \quad y \approx -\frac{\sinh(U_p)}{\cosh^3(U_p)}r^2 - \tanh(U_p). \quad (5)$$

Equation (5) is the simple analytic form for the tunneling factor γ that we desire; its simplicity suits it to implementation in simulators such as SPICE. Further simplification of γ is possible by dropping the factor of 1, such as would be appropriate in cases where the tunneling significantly exceeds the thermionic emission current.

III. DISCUSSION

Examination of (1) shows that J_C is proportional to γ. Thus, the quantum mechanical nature of the CBS directly manifests itself, through γ, in the determination of J_C. This result reaffirms the statement that modelling the current transport in the CBS is of paramount importance to the understanding of abrupt HBTs.

Further consideration of the subsidiary equations for U_{max} and U_p reveals, the following general traits: as U_p increases from 0 towards infinity, U_{max} tends from 1 towards zero, and tunneling becomes increasingly dominant over thermionic emission; as N_D increases, or ε_n decreases, the width of the CBS decreases and U_{max} becomes smaller, showing that tunneling is increasing; as m_n^* decreases the probability of tunneling should increase, as is confirmed by the associated reduction in U_{max}; finally, in the limit as $\hbar$ goes to zero, the system should evolve to a state that is purely describable by classical mechanics, and it is found that U_{max} goes to 1, which indicates that there is indeed no tunneling. Therefore, the general traits of the emission flux, as presented, follow physical expectations.

Before presenting the final form of J_C, with γ from (5) included, the error associated with the form given by (5) is illustrated via the plots shown in Fig. 3. Note that as T increases γ decreases; this is expected as more carriers can be thermally excited at higher T, and thus tunneling becomes less important relative to thermionic emission. The discrepancy between (5) and the exact form (2) at first decreases with bias. This is because the exact lower limit of integration (used in (2) and shown in Fig. 2) tends to the limit of $-\infty$ as V_{BE} increases. This improvement in accounting for the emission flux at energies below the maximum U_{max}, more than outweighs the discrepancy at higher energies which increases with bias (see Fig. 2 and note the placement of the upper limit). This latter discrepancy amounts to an inclusion of the thermionic-emission flux in the tunneling integral, i.e. a double-counting in γ of the emission flux density above the peak of the CBS. It is this double-counting that results in the increasing discrepancy between (5) and the exact form (2) at high biases.

The final form for J_C is achieved by substituting (5) into (1) to give:

$$J_C = q\upsilon \frac{n_{i,p}^2}{N_A} \sqrt{\frac{4\pi \sinh(U_p)\, U_p E_c(0^-)}{\cosh^3(U_p)\, kT}}\, e^{\frac{-\Delta E_{n0}}{kT}}\, e^{q\frac{N_{rat}V_{bi}}{kT}\left(1-\frac{\tanh(U_p)}{U_p}\right)}\, e^{\frac{N_{rat}\tanh(U_p)}{U_p}\frac{qV_{BE}}{kT}} . \qquad (6)$$

Examination of (6) shows that J_C is basically proportional to $\exp[qN_{rat}\tanh(U_p)V_{BE}/(U_p kT)]$ (this is achieved by disregarding the small variation with bias of $E_c(0^-)$ in the square root term of (6)). Thus we find the customary exponential relationship between J_C and V_{BE} that is found experimentally. However, we now realise that the injection index n is not 1 (as is given by Shockley boundary conditions) but is instead given by $n=U_p/(N_{rat}\tanh(U_p))$. For the device considered in Section II this gives $n = 1.13$, which is almost exactly what is found experimentally. In fact, the slightly larger values found for n experimentally can be accounted for by the bias dependence of the term in the square root of (6).

IV. CONCLUSION

We have achieved a tractable, analytic formulation for both the tunneling factor γ (5) and the transport current J_C (6), and both formulations are suitable for implementation in simulators such as SPICE. Finally, due to the analytic nature of these results, clear physical insight into the connection between material parameters and device operation is obtained, e.g., the new formulation for the injection index of J_C.

V. REFERENCES

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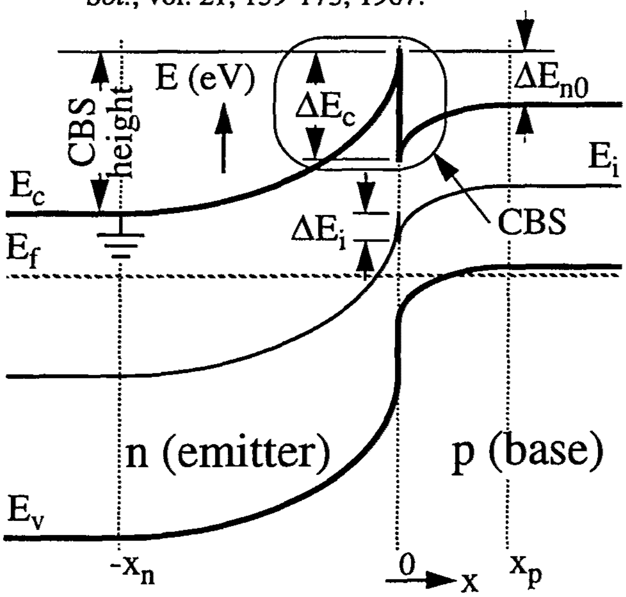

Fig. 1. Equilibrium Band Diagram for the abrupt emitter-base junction of a NPN HBT with a type I band alignment. Note: the reference potential is $E_c(x = -x_n) \equiv 0$.

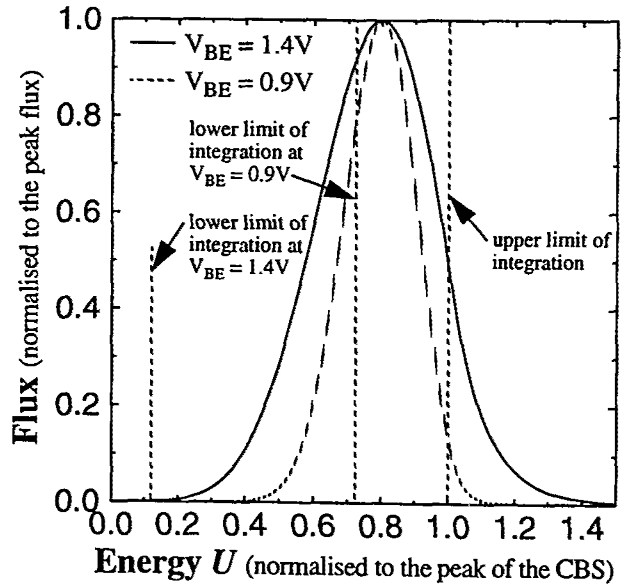

Fig. 2. Emission flux density for an $Al_{.3}Ga_{.7}As/GaAs$ abrupt HBT at two different forward biases. The material parameters are given in Section II.

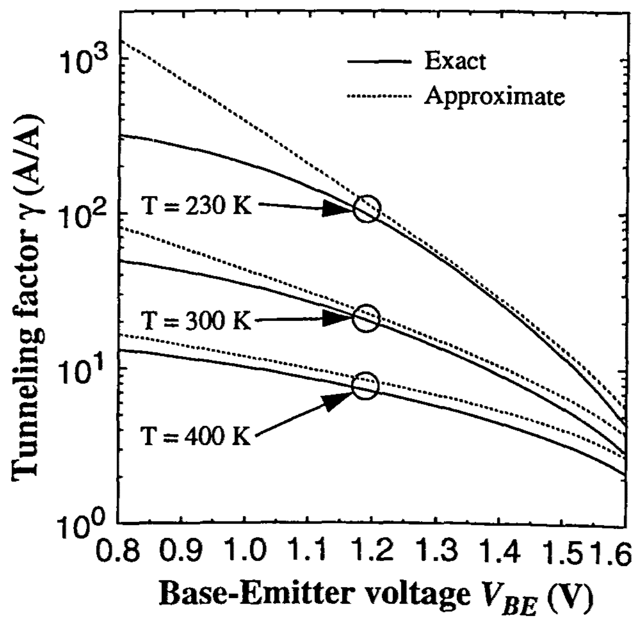

Fig. 3. Plot of the exact (2) and approximate (5) γ versus applied potential for the abrupt HBT detailed in Section II. Note: as the temperature increases γ decreases due to the expected increase in thermionic emission.