INCLUSION OF VISCOUS EFFECTS IN THE HYDRODYNAMIC MODELING OF ULTRASMALL SILICON DEVICES

Ting-wei Tang and Joonwoo Nam
Department of Electrical and Computer Engineering
University of Massachusetts, Amherst MA 01003

Abstract

The effect of viscosity on the modeling of the second moment energy tensor $\hat{U}$ was investigated for the narrow base width Si BJT's. The overestimation of velocity overshoot occurring at the base-collector junction often predicted by the conventional hydrodynamic (HD) models can be reduced if a viscous term is included in the modeling of $\hat{U}$. However, the overall effect of including the viscous effect in the HD model on the device characteristics is yet to be investigated.

I. INTRODUCTION

In recent years, many advanced HD models have been developed for the simulation of deep submicron MOSFET's or ultranarrow base width Si BJT's. One of the objectives is to accurately predict the velocity overshoot phenomenon. Although much progress has been made, most of currently existing HD models tend to overestimate the velocity overshoot [1]. This overestimation may become worse, for example, as the base width of BJT's reduces to 0.1 μm or less. In this paper, we propose to include a viscous term in the modeling of second moment energy tensor $\hat{U}$ as a possible mechanism for reducing the velocity overshoot.

II. MODELING OF $\hat{U}$

The macroscopic equation governing the velocity (or momentum) can be rigorously derived by taking the first moment of the Boltzmann transport equation and integrating over the entire momentum space. In the momentum transport equation so obtained, a second-order moment $\hat{U} = \left\langle \vec{v}\hbar\vec{k} \right\rangle$ sometimes called energy tensor or momentum flux tensor appears as a result of taking the moment. The simplest model for $\hat{U}$ is $k_B T_e \hat{1}$ where $\hat{1}$ is a unity tensor. A slightly more advanced model is given by $\hat{U} = \frac{1}{2}m_c\vec{V}\vec{V} + k_B T_e \hat{1}$ [2] where $\vec{V} = \langle \vec{v} \rangle$ and m_c is the effective mass. A recent Monte Carlo (MC) calibrated model gives $\hat{U} = \vec{V}\vec{P} + [\frac{2}{3}W + u(W)]\,\hat{1}$ where $\vec{P} = \left\langle \hbar\vec{k} \right\rangle$, $W = \left\langle \varepsilon(\vec{k}) \right\rangle$ and $u(W)$ is an empirical expression fitted to MC data [3].

In all the models for $\hat{U}$ mentioned above, none of them contains a viscous term. It is well known in gasdynamics that the Navier-Stokes equation includes a viscous term which is proportional to the gradient of velocity. For example, assuming a parabolic band structure and a heated displaced Maxwellian for the distribution function under the homogeneous field, an iterative solution for the inhomogeneous field yields a second-order viscous term for $\hat{U}$, e.g., in one dimension, $(U_{xx})_{visc.} = -\bar{\eta}\tau_c m_c V_x^2 \frac{dV_x}{dx}$, where τ_c is the collision relaxation time and $\bar{\eta}$ is a dimensionless coefficient of order unity. A recent analysis based on extended thermodynamics [4] gives $(U_{xx})_{visc.} = -\frac{4}{3}k_b T_e \tau_\sigma \frac{dV_x}{dx}$

where τ_σ is the viscous stress relaxation time. In this work, we examine whether or not such a second-order effect should be included in the modeling of $\hat{U}$, and if so, what the approximate value of $\bar{\eta}$ based on the MC data is.

III. MODEL CONSISTENCY TEST

We focus our investigation on ultrasmall base width Si BJT's since the gradient of velocity near the base-collector junction is known to be very large in these devices. A one-dimensional BJT as shown in Fig.1 was used as a prototype for model testing.

Our MC model consistency test proceeds as follows [1]. First, we solve a HD model for the BJT to obtain an electric field profile within the device. Using this electric field, we perform a fixed-field MC particle simulation to obtain all the necessary macroscopic quantities such as $\vec{V}$, $\vec{P}$, W, $\hat{U}$, etc. To test the consistency of a model, say, $\hat{U} = \frac{2}{3}W\hat{1}$ or in one dimension $U_{xx} = \frac{2}{3}W$, we plot U_{xx} vs W using the position in the device as an implicit parameter. Since the average electron energy W always increases from its thermal equilibrium value W_o to a maximum and then falls back to W_o inside the device, the plot of U_{xx} vs W will trace the horizontal axis twice. If U_{xx} vs W traces a curve without a "hysteresis" loop, then U_{xx} is indeed a single-value function of W and $U_{xx} = \frac{2}{3}W$ is a consistent (good) model. In general, U_{xx} vs W will trace a loop and the larger the loop, the poorer the model. Thus, if U_{xx} is modeled as $U_{xx} = V_xP_x + \frac{2}{3}W + u(W)$, we should expect the plot of $(U_{xx} - V_xP_x)$ vs W to exhibit a very small hysteresis since $\frac{2}{3}W + u(W) = U_{xx} - V_xP_x$ is supposed to represent a single value function of W.

In the following,we compare two sets of MC data. One is without the viscous effect, i.e., the plot of $(U_{xx} - V_xP_x)$ vs W. Another, with the viscous effect included, is the plot of $(U_{xx} - V_xP_x + \bar{\eta}\frac{m_c}{q}\mu V_xP_x\frac{dV_x}{dx})$ vs W where μ is the electron mobility and $\bar{\eta}$ is a dimensionless parameter to be adjusted until the loop disappears. As shown in Fig.2, when $(U_{xx} - V_xP_x)$ is plotted against W, a large "hysteresis" loop exists in the high energy range $0.1eV < W < 0.3eV$ and a small one in the low energy range, $0.04eV < W < 0.1eV$. The relatively large loop is a result of a rather crude approximation for $\hat{U}$ by incorporating a tensorial component equal to $\vec{V}\vec{P}$ in the modeling of $\hat{U}$ [3]. This energy range is beyond the range in which velocity overshoot takes place. By including a viscous term we hope that the small hysteresis loop in the energy range $0.04eV < W < 0.1eV$ can be substantially reduced. This is indeed the case as when $(U_{xx} - V_xP_x + \bar{\eta}\frac{m_c}{q}\mu V_xP_x\frac{dV_x}{dx})$ is plotted against W, the small loop virtually disappears if $\bar{\eta}$ is chosen to be approximately $\frac{4}{3}$ (see. Fig.2).

Next, we compare the modeled $\hat{U}$ with the input data from the MC and the $\hat{U}$ directly obtained from the MC. In order to compare the two models, $U_{xx}^{(1)} = V_xP_x + \frac{2}{3}W + u(W)$ and $U_{xx}^{(2)} = V_xP_x + \frac{2}{3}W + u(W) - \bar{\eta}\frac{m_c}{q}\mu V_xP_x\frac{dV_x}{dx}$, we can input the MC data for V_x, P_x, W, etc. on the right-hand side of $U_{xx}^{(1)}$, $U_{xx}^{(2)}$ and compare both of them with the $U_{xx}^{(MC)}$ as a function of position. The comparison between the modeled U_{xx}'s with and without the viscous term and that of MC data is shown in Fig. 3. At first glance, the difference between the two models appears very small. However, since the electron velocity is given by $\vec{V} = -\frac{\mu}{q}\left[q\vec{E} + \hat{U}\cdot\nabla(\ln n) + (1-\lambda_p)\nabla\cdot\hat{U}\right]$ [3], it is easy to see that V_x is very sensitive to the slope, $\frac{dU_{xx}}{dx}$. Based on the MC consistency test [1], the model accuracy for predicting V_x is compared in Fig. 4 for the two U_{xx} models. The effect of the viscosity is seen to reduce the velocity peak in a better agreement with the MC result and is also confirmed by other investigation [5]. We also performed a similar study for $n^+ - n - n^+$ structures with active region less than 0.1 μm. Although inclusion of the viscous term with $\bar{\eta} \approx 2$ makes a more accurate representation for U_{xx}, its effect on the velocity overshoot is not as significant as in the BJT's.

IV. CONCLUSION

From the MC consistency test, it seems to suggest that a more accurate modeling for $\hat{U}$ should include a viscous term. In one-dimensional case, we found that $U_{xx} = V_x P_x + \frac{2}{3}W + u(W) - \bar{\eta}\frac{m_c}{q}\mu V_x P_x \frac{dV_x}{dx}$ where $\bar{\eta} \approx \frac{4}{3} \sim 2$ fits best with the MC data. This range of coefficient for $\bar{\eta}$ also agrees with values predicted by the others [4]. Since the emphasis of this work is to see whether it is necessary to include the viscous effects in the simulation of narrow base with Si BJT's; the solution to the full set of HD equation including the viscous term was not attempted. A rigorous numerical solution to such a system of equations requires a solution strategy different from the conventional one. This is because when the viscous term is included, the order of differential equation representing the momentum conservation is raised by one and the added viscous term represents a singular perturbation in the limit of vanishing viscosity [5]. Although it is yet to be confirmed, we believe that the viscous effect may be important in modeling of advanced BJT's with ultranarrow bases because the gradient of velocity at the base-collector junction of such devices is usually very large.

ACKNOWLEDGEMENT

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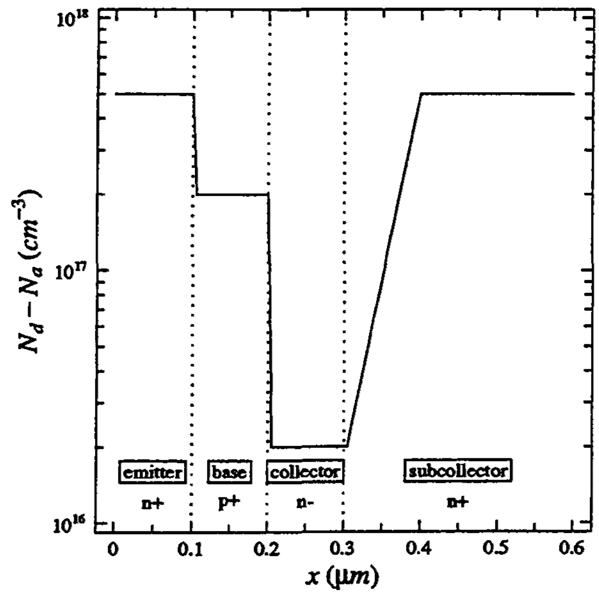

Figure 1: Doping profile of an *npn* BJT.

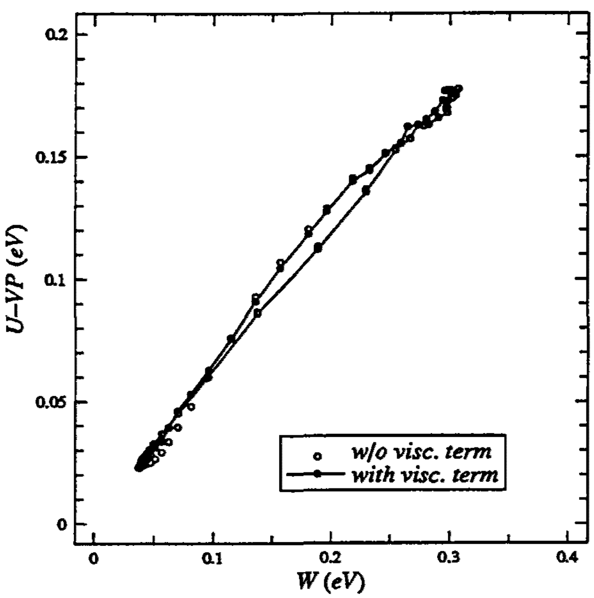

Figure 2: $U - VP$ with and without the viscous term as a function of the average energy W.

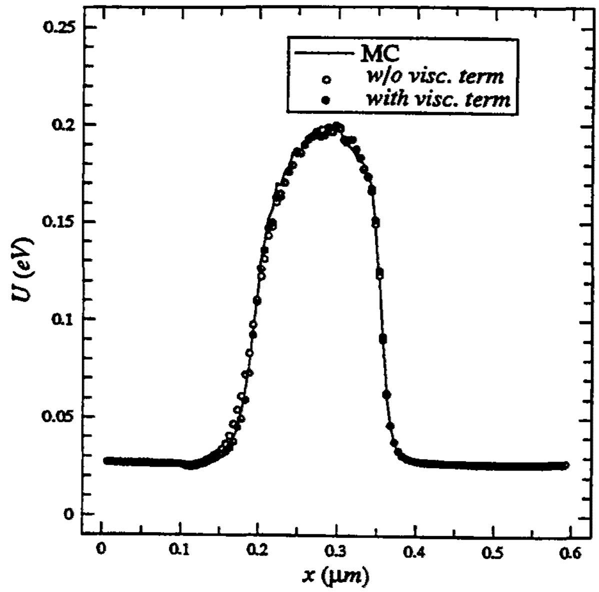

Figure 3: Modeling of U with and without the viscous term as a function of position.

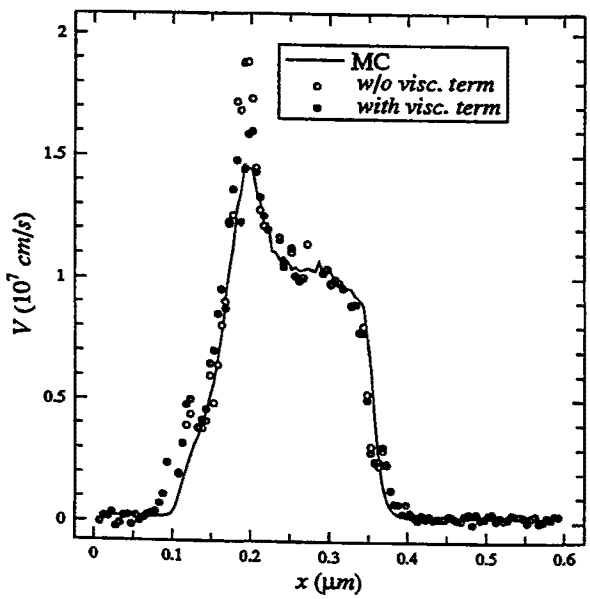

Figure 4: Modeling of V with and without the viscous term as a function of position.