An Equivalent Circuit Model and Distribution-Function Theory of the High-Frequency Behavior of Quantum-Based Devices

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Abstract
The high-frequency response of resonant tunneling devices (RTD), subjected to time-dependent signal, is considered in both stable and unstable situations when operating in the negative differential resistance (NDR) region, based on the phase-space distribution function formalism. In the stable case, the equivalent-circuit approach (ECA) is shown to characterize the complex high-frequency behavior of a RTD's response to small a.c. signal. The ECA is found to be very useful in resolving the various outstanding controversies concerning the dynamical quantum transport behavior of RTD. For the unstable case, nonperturbative approaches are outlined.

I. Introduction

There is much confusion in the literature concerning the high-frequency behavior of resonant tunneling devices (RTD). The numerical simulation of Frensley [1], using the Wigner distribution-function transport equation (WDFTEQ), reveals a capacitive behavior at lower frequencies, eventually changing into an inductive behavior at higher frequencies. In contrast, the numerical simulation of Kluksdahl, et al. [2] using a similar WDFTEQ approach reveals an inductive behavior at low frequencies, changing into a capacitive behavior at intermediate frequencies, and eventually changing back into a (somewhat) inductive behavior at high frequencies. Both of these simulations do agree concerning the real part of the admittance, namely, it is negative at lower frequencies and becomes positive at high frequencies. Their results also concur at high frequencies by having a cutoff in the admittance. Until now, the discrepancy in the reactive behavior cited above has remained unresolved.

So far, there are no other serious work on the subject of characterizing the high-frequency response of RTD to a.c. signal, although there have been a number of attempts to analyze the high-frequency behavior of nanometric structures. These are attempts to extend the Landauer-Büttiker viewpoint, which calculates conductance from transmission coefficients, to high frequencies. The gross deficiencies of all these attempts have been discussed by Landauer [3]. There are also attempts, for example by Fu and Dudley [4], which employ the machinery of the linear response theory, however this particular approach is plagued by its inability to treat far-from-equilibrium operating bias conditions, of interest to the nanoelectronics community. More recent results of Cai and Lax [5] employ a nonperturbative time-dependent Green's function approach to compute the behavior of an electron incident from an energy channel on a double-barrier structure. Their results reveal the presence of intrinsic oscillations in response to a voltage pulse across the negative differential resistance (NDR) region, similar to the result obtained by Buot and Jensen [6], particularly when self-consistency was switched off in their WDFTEQ calculations. The results of Cai and Lax are however limited to single-channel and nonselfconsistent contributions.

The purpose of this paper is twofold: to show that a coherent and unified understanding of the high-frequency behavior of RTD arises from the equivalent-circuit model of Buot and Jensen (BJ) [7], to be referred here as the BJ model (originally proposed to study the nonlinear aspects of RTD which arises from the selfconsistent treatment of WDFTEQ), and to point out a proper way to a nonperturbative theoretical framework for analyzing the high-frequency behavior of resonant tunneling devices within the quantum distribution-function approach. It is worthwhile to stress the first point here since there have been misleading statements in the literature claiming that no understanding exists for quantum transport at nonzero frequencies. Also there is a need to clarify a statement made in the literature which strongly implies that no LRC circuitry can simulate the complicated frequency dependence of the admittance in the linear-response regime, particularly since this statement is made without further reservations. We will here show that indeed the two earlier numerical simulations mentioned above [1,2] may be recast in terms of the BJ model in two separate regimes in the BJ circuit parametrization, operating in the NDR region. Thus, the BJ model is found to include the high-frequency behavior of RTD.

II. Equivalent-Circuit Model

The BJ model is derived from the selfconsistent quantum distribution-function (QDF) transport simulation, with four independent equivalent-circuit parameters, namely, series resistance, R, negative conductance, G, quantum inductance, L, and capacitance, C. The RTD simulated by BJ, with fixed bias applied in the NDR, is characterized by the following inequalities for the circuit parameters: $R\,|G| > 1$ and $RC > L|G|$. These are conditions for the presence of oscillatory behavior and instability of the operating point in the middle of the NDR region [7] leading to the intrinsic bistability and hysteresis in the current-voltage characteristics,which was the focus of the original BJ model. In contrast, we now find that the RTD simulated by Frensley [1] and Kluksdahl, et al. [2] are obtained if the following inequalities are obeyed: $R\,|G| < 1$, and $RC < L|G|$, along with further conditions to be specified in what follows.The above are conditions for the nonoscillatory and stable operation in the middle of the NDR region allowing for the applicability of linear-response approximation [8]. They also imply that the series resistance is quite small compared to that of the RTD simulated by BJ (note that the RTD simulated by BJ, although having approximately the same feature sizes for the double-barrier region, differs in having an undoped buffer layer on both sides of the double barrier, which may also affect the value of the capacitance relative to those of Refs. [1] and [2], and a longer computational box length, which together introduces a larger R). Moreover, the RTD simulated by Frensley [1], without taking self-consistency into account, differs from that simulated by Kluksdahl et al. [2], which takes self-consistency into account, by the following inequality: $L|G| < C/|G|$ for Frensley [1], whereas $L|G| > C/|G|$ for Kluksdahl et al [2]. These imply a larger quantum inductance for the RTD of Ref. [2] as compared to that simulated in Ref. [1], where the residual small inductance is mainly due to the quantum nonlocality [8]. Further, note that Ref. [2] uses wider barrier widths than those used in Refs. [1] and [7]. Thus the BJ circuit model serves not only to clarify the origin of the discrepancy of the results of the two numerical simulations [1,2], but also to bring together the various dynamical aspects of RTD in terms of WDFTEQ.

In addition to the quantum inductance, in parallel with the capacitance, found by BJ for fixed bias in the NDR region, it is important to take also into account effects arising from the electron kinetics in calculating the RTD response to high-frequency a.c. signal. These effects are often referred to as due to the electron inertia, i.e., it takes time for the electron to be accelerated and decelerated, typically causing the current to lag in time behind the electric field. These inertial effects, although always present, are negligible at low frequencies. It is taken into account in our high-frequency equivalent circuit model of RTD by adding another inductance, ℓ, (ℓ is typically an order-of-magnitude smaller than the quantum inductance, L) in series with R, outside of the two-branch circuit of L, G, and C. As will be shown in the following figures, this additional "inertial inductance", ℓ, serves to cut off the RTD response at very high frequencies.

We have calculated the admittance of RTD to a small a.c. signal using the above-mentioned high-frequency equivalent circuit model. The different circuit parameters are estimated by adjusting the values obtained by BJ to conform with the constraints enumerated above appropriate to the different RTD simulated. The results are displayed in Fig. 1 for the RTD parameters appropriate to the device simulated by Frensley [1], and Fig. 2 for the RTD parameters appropriate to the device simulated by Kluksdahl et al. [2] (note that the inductive part of the admittance is plotted with a positive scale in Ref. [2]). Observe that the characteristic features of the results of their numerical simulations, mentioned at the beginning, are well reproduced by the present results, obtained simply by using the high-frequency equivalent-circuit model. Indeed, these results lend further support to the accuracy of the BJ equivalent-circuit model of RTD at low frequencies [7], as well as confirm the validity of the present high-frequency equivalent circuit model of RTD which incorporates a series-inertial inductance. These results also serve to invalidate the claim [4] made without any reservation, that no LRC circuit can simulate the complicated frequency dependence of the admittance in the linear-response regime.

III. Nonperturbative Approaches for Unstable NDR Operation

For unstable operation in the NDR region, nonperturbative approaches are more appropriate. Here, a new time-dependent transformation of phase space is found which transform the quantum distribution (QDF) transport equation to the same form in the absence of time-dependent signal. This is shown to be a very powerful approach in revealing the nature of the nonperturbative response to a small a.c. signal. This time-dependent transformation is useful when the applied time-dependent electric field is assumed to be position-independent. The general formulation of the QDF transport equation in the presence of space and time-dependent potential is given by one of the authors [6,9]. For high-frequency signals applied at the drain terminal of an RTD, following the conventional procedure, we assume a time-dependent but position-independent applied perturbing field within the device. From the general formula given in Refs. [6,9], one arrives at a resulting QDF transport equation in response to this signal. We show that the resulting QDF transport equation can be transformed into the form of the equation obtained earlier in the absence of the perturbing time-dependent field, through a time-dependent transformation of the phase-space. This transformation of the equation however makes the double-barrier potential to be time-dependent. With this simplification, the application of the numerical technique of Ref. [6] with known time-dependent matrix for the double-barrier potential operator, with time as a parameter, becomes feasible. The derivation of the transformed and simplified new QDF transport equation, and the method of the numerical implementation for calculating the nonperturbative response to an a.c. signal will be discussed in a forthcoming paper.

In a more general and realistic situation, an applied time-dependent voltage at the drain will selfconsistently lead to a time-dependent and position-dependent potential inside the device. For this general situation of the unstable case, a nonperturbative approach based on the structure of phase space is proposed. We introduce two different representations of quantum transport, namely, the Liouville representation, and, the phase-space fluid representation. The QDF is

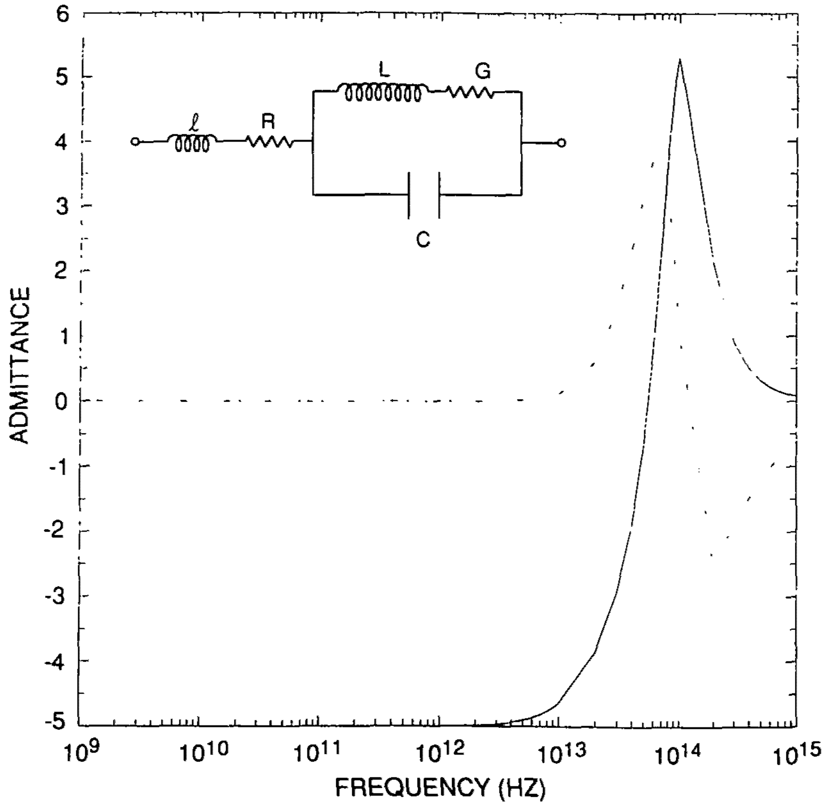

Fig. 1 - Real (solid curve) and imaginary (dashed curve) parts of the admittance (in units of 10^7 S/cm^2) for the circuit parameter combination that corresponds to the RTD simulated by Frensley [1]: $R|G| < 1$, $RC < L|G|$, and $L|G| < C/|G|$. The values used are: $R = 0.2 \times 10^{-7}$ Ω cm^2, $G = -2.5 \times 10^7$ S/cm^2, $C = 6.0 \times 10^{-7}$ F/cm^2, $L = 0.8 \times 10^{-2}$ H/cm^2, and $\ell = 0.2$ L.

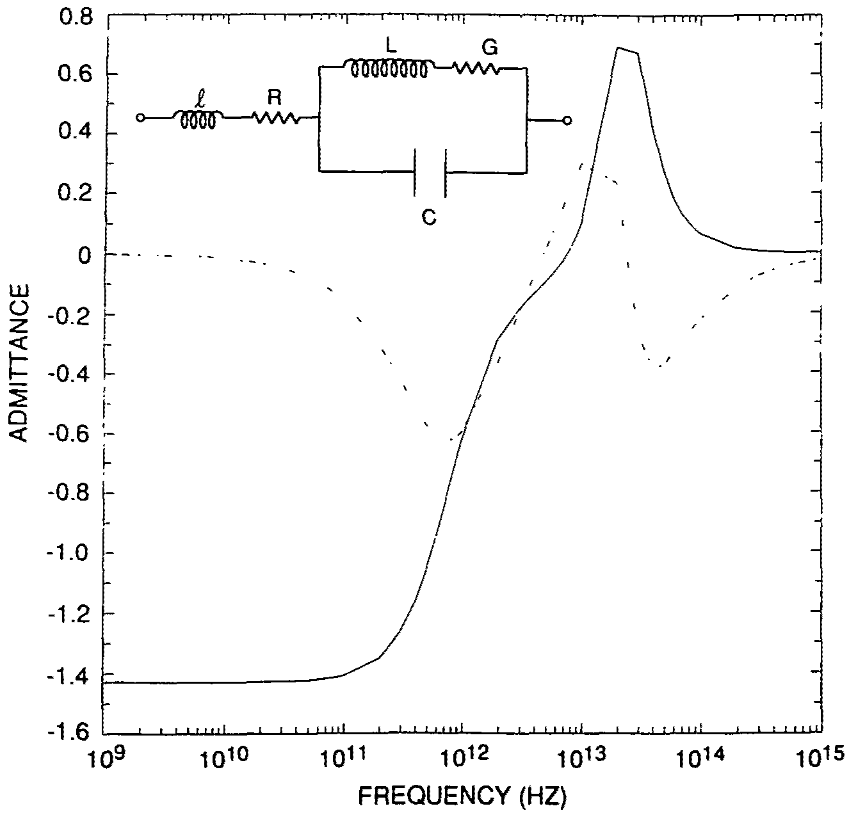

Fig. 2 - Real (solid curve) and imaginary (dashed curve) parts of the admittance (in units of 10^7 S/cm^2) for the circuit parameter combination that corresponds to the RTD simulated by Kluksdahl et al. [2]: $R|G| < 1$, $RC < L|G|$, $L|G| > C/|G|$. The values used are: $R = 1.3 \times 10^{-7}$ Ω cm^2, $G = -0.5 \times 10^7$ S/cm^2, $C = 0.4 \times 10^{-6}$ F/cm^2, $L = 9.0 \times 10^{-20}$ H/cm^2, and $\ell = 0.05$ L.

solved numerically in the same fashion as described and successfully implemented in the existing literature, but using time-dependent bias. The QDF solution has inherent undesirable features for studying the dynamics of phase space, which can be eliminated by a special post-processing. This post-processing yields a smoothed-out QDF, the positive definite Husimi distribution, and allow us to study the structure of quantum trajectories in phase space. It is suggested that the use of Husimi distribution enables a microscopic dynamical viewpoint of ECA as well as shed further light on the dynamical nature of the quantum inductance. The full details of this approach will be discussed in another paper.

IV. Concluding Remarks

In conclusion, the utility of the QDF approach for understanding the high-frequency behavior of quantum-based devices and in deducing the equivalent circuit model of RTD have been demonstrated here. It is worthwhile to reiterate that the equivalent circuit model of RTD is subtle and in the form presented here elucidates the two earlier simulations [1,2] as special cases of the model presented here. Thus the discrepancy in the simulated reactive behavior mentioned in the opening paragraph is resolved. Moreover, the exact role of electron inertia in the high-frequency behavior of RTD is clarified. For cases when the linear response approximation breaks down, nonperturbative schemes are outlined.

Acknowledgment

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