NUMERICAL STUDY OF QUANTUM MAGNETOTRANSPORT IN DISORDERED NON-ADIABATIC CONSTRICTIONS

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Abstract

We have numerically calculated the magnetic field dependence of electron transmission probabilities through a non-adiabatic constriction containing an elastic scatterer. The transmission probabilities are found directly from the scattering matrix of the constriction and the elements of this matrix are evaluated by the real space mode matching technique. This technique requires knowledge of the wavefunctions of the various modes or subbands (both propagating and evanescent) in the constriction and these are found by solving the Schrödinger equation using a finite difference scheme. From the transmission probabilities, we compute the 2-probe linear response conductance as a function of a magnetic field. We observe pronounced negative magnetoresistance which agrees with past experimental observations.

Electron transmission through narrow ballistic constrictions subjected to an external magnetic field has been a topic of extensive experimental and theoretical study for the last few years [1,2]. The theoretical studies [2] have concentrated on computing the transmission from a Green's function using the Fisher-Lee formula [3]. The effect of the magnetic field is incorporated in the Green's function through a Peierl's phase factor. However, Peierl's substitution has recently been criticized [4] on the ground that this can lead to both quantitative and qualitative errors in the calculation of energy levels of quasi two dimensional electron gases. Therefore, the Green's function technique, which is used presumably for its relative simplicity and computational ease, is not a reliable technique, especially when the magnetic field is quite strong.

In this paper, we have calculated electron tranmission through a narrow constriction containing an elastic scatterer using a completely different technique. The transmission is calculated directly from a *scattering matrix* describing electron propagation through the disordered constriction in the presence of a magnetic field. This scattering matrix can be computed *exactly* for a δ-scatterer. Reference 5 describes how this matrix is calculated in the absence of a magnetic field and reference 6 has extended it to the case when a magnetic field is present. Owing to space limitations, we will present only the basic features of this method here; for further details, the reader is referred to Refs. 5 and 6.

The scattering matrix of a disordered constriction subjected to a magnetic field is found by first breaking up the structure into a number of sections - one containing the junction between the wide and narrow regions (plane $y - y'$ in Fig. 1) and the others containing the scatterers. A scattering

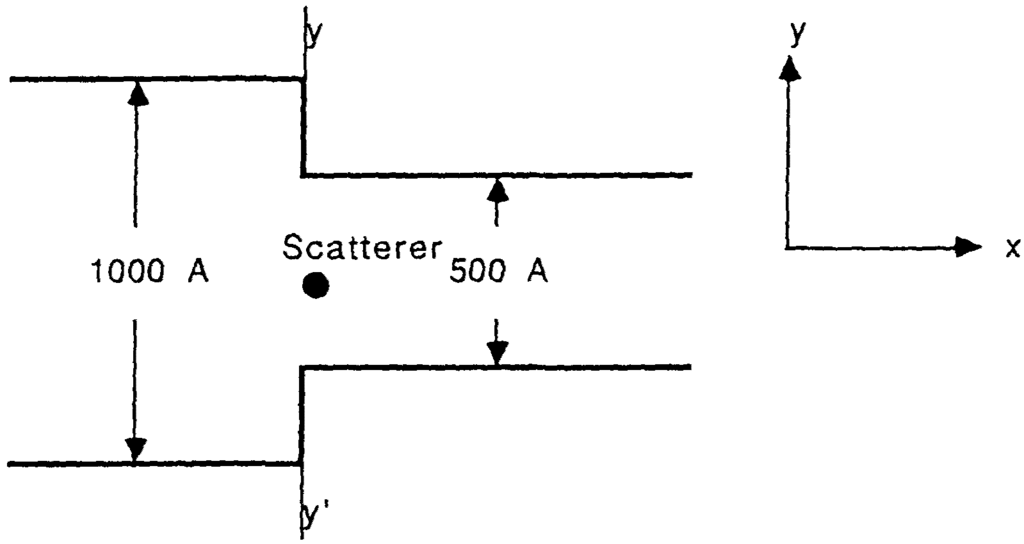

Figure 1: A constriction containing an elastic scatterer and subjected to a magnetic field in the z-direction.

matrix for each such section is then found by relating the amplitudes of the reflected modes (channels) to those of the modes incident on that section. Both propagating and evanescent modes must be considered. This requires matching wavefunctions of the modes (i.e. wavefunctions of the various subbands) and their first derivatives across the section. We accomplish this by the *real space mode matching technique* [7] as opposed to the *k-space mode matching technique* [8] which is numerically not suitable for acheiving quick convergence when the modes are not orthogonal (they are *not* orthogonal in the presence of a magnetic field). The wavefunctions of the modes in any section are calculated numerically using a finite difference scheme for solving the Schrödinger equation in a magnetic field [9]. Finally, the overall scattering matrix is found by cascading the individual scattering matrices according to the Redheffer rule [10] and from this overall matrix, the transmission through the entire structure is obtained directly.

In Figs. 2-4 we plot results for a constriction whose wide region has a width of 1000 Å and the narrow region has a width of 500 Å. The δ-scatterer is placed right at middle of the junction as shown in Fig. 1; it represents an attractive impurity. The material is assumed to be GaAs and the Fermi wavevector is 10^6/cm. Two subbands are occupied in the wide region and only one in the narrow region. We calculate the transmission amplitudes of electrons incident from both occupied subbands (modes) in the wide region into the only occupied subband (propagating mode) in the narrow region as well as into the two lowest unoccupied subbands (evanescent modes) in the latter region. These amplitudes are computed as a function of the magnetic flux density and the results are shown in Figs. 2 and 3. Note that the magnetic field dependences of the transmission amplitudes are *non-monotonic*.

Finally, we also calculate the two-probe linear response magconductance $G(B)$ as a function of the magnetic flux density B from the Landauer-Büttiker formula. The magnetoconductance result is shown in Fig. 4. It is interesting to note that even though the magnetic field dependences of the transmission amplitudes are strongly non-monotonic, the conductance is almost a monotonic function of the magnetic field. The constriction exhibits pronounced negative magnetoresistance which agrees with experimental observations reported by a number of researchers in the past [11].

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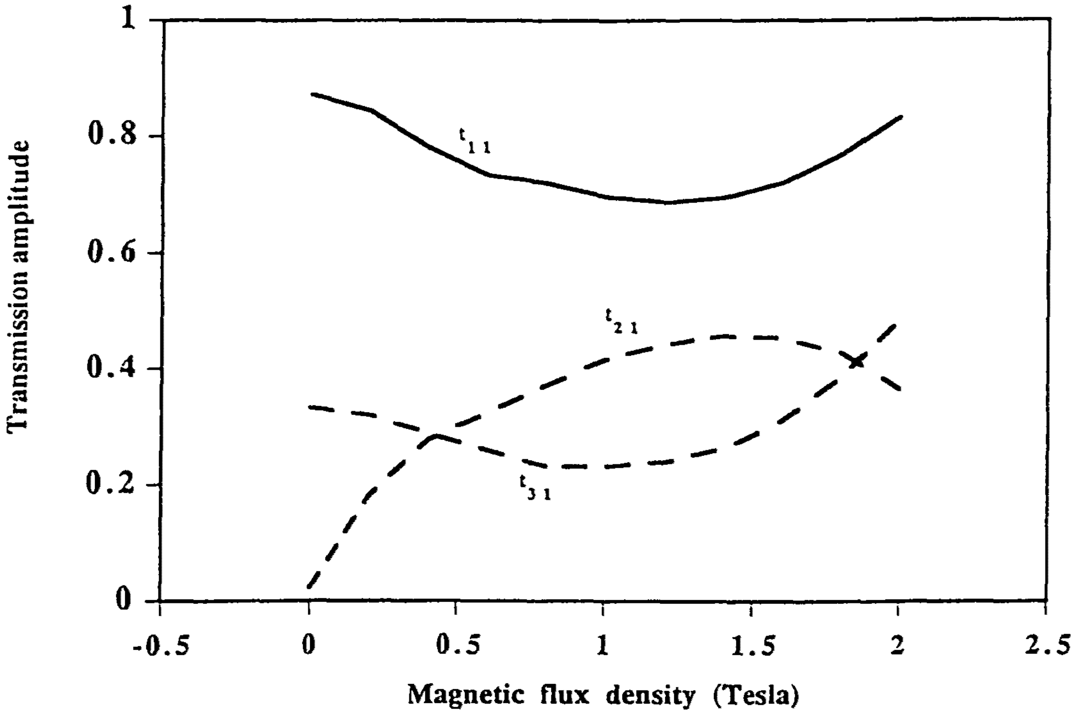

Figure 2: Magnetic field dependence of the transmission amplitude t_{i1} describing propagation from the lowest subband in the wide region to subband i in the narrow region. The solid curves correspond to the case when subband i is occupied (propagating mode) and the broken curves are for the case when subband i is unoccupied (evanescent modes).

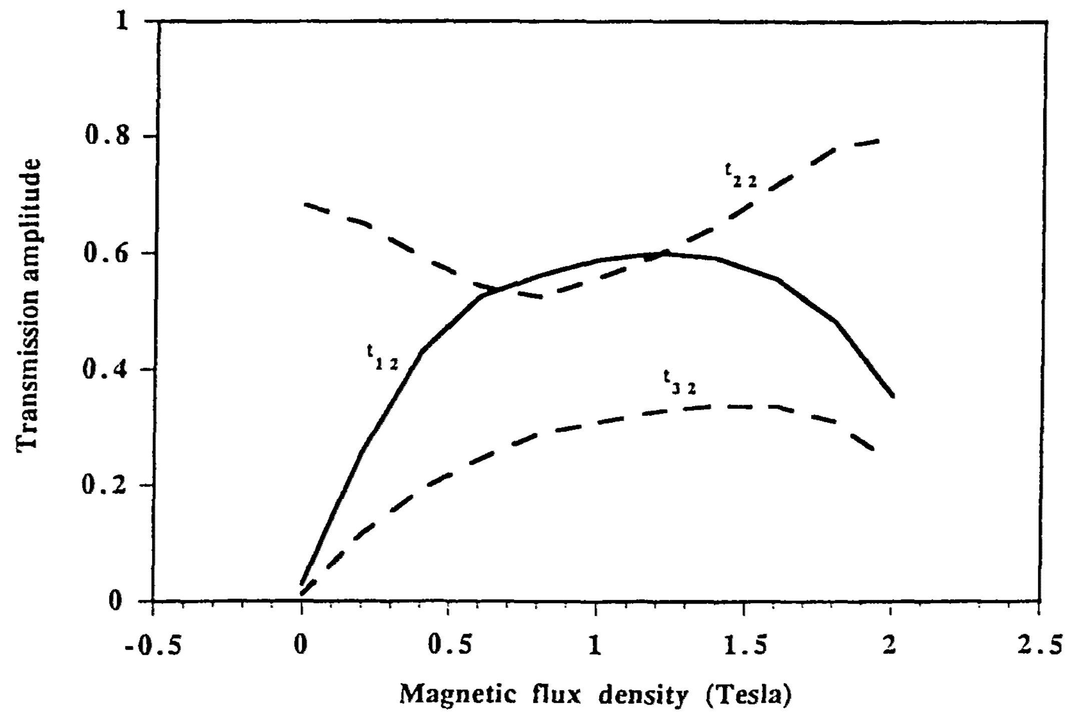

Figure 3: Magnetic field dependence of the transmission amplitude t_{i2} describing propagation from the second lowest subband in the wide region to subband i in the narrow region. The solid curves correspond to the case when subband i is occupied (propagating mode) and the broken curves are for the case when subband i is unoccupied (evanescent modes).

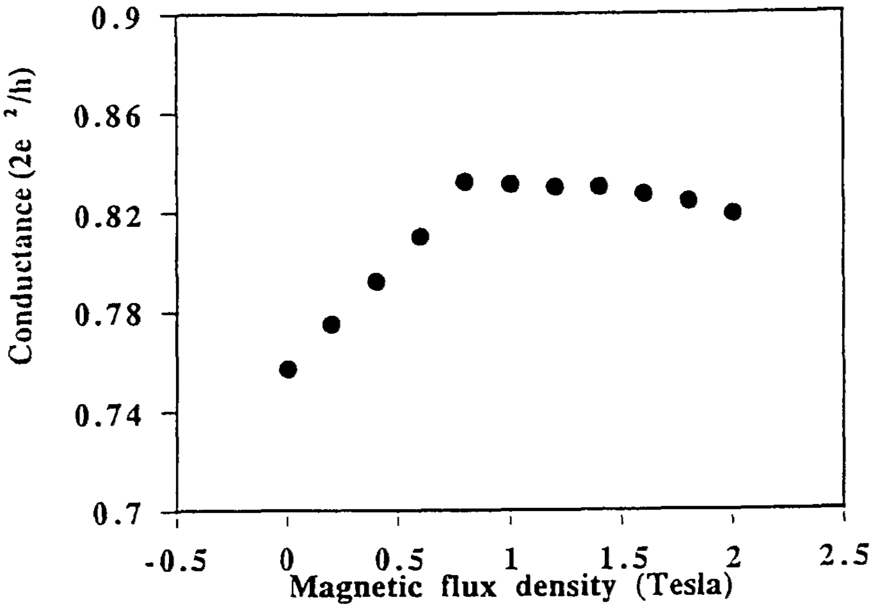

Figure 4: Two-probe linear response conductance of the constriction in Fig. 1 as a function of magnetic flux density.

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