NUMERICAL STUDY OF MESOSCOPIC STRUCTURES WITH MULTIPLE CHANNELS

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Abstract

The conductance of parallel mesoscopic channels of electron propagation (electron waveguides) is investigated, and criteria are given which establish whether or not conductances of two channels add, as they do for classical conductors. We find that the laws of addition of conductance are a consequence of the properties of the interfaces at which the electron waveguides divide, and can be most conveniently understood by studying the transmission and reflection coefficients at these interfaces.

1. Introduction

The possibility of fabricating arrays of parallel mesoscopic constrictions has raised the question whether their conductances simply add, as they do for classical conductors or whether interference effects play an important role if the distance between them is smaller than the phase coherence length. Avishay *et al.*[1] and Castaño and Kirczenow[2] have studied this problem, and have obtained the unexpected result that the conductances of parallel channels add in most cases as the ones of classical resistors. An experimental verification of such additivity has been given by Smith *et al.*[3], who have measured the conductance of two point contacts very close to each other.

We present in this paper a detailed study of this law of addition and its failures. This study is based on the properties of interfaces at which the electron waveguide bifurcates into two channels. We derive the basic principles which lead to the addition of the conductances and to deviations from the laws of addition. All the structures included in our research are bounded by infinite (hard) potential walls and can be decomposed into rectangular sections, so that, for the solution of the Schrödinger equation, mode matching is the method of choice[4,5], and can be implemented very efficiently.

2. Single interface

Consider a waveguide which partitions into two channels, as shown in Fig. 1(a): for mode i impinging from the upper left channel ("driven channel") the mode matching equations at the interface, when projected on appropriate sets of transverse modes [6], read:

$$\frac{1}{\sqrt{k_i}}\int_0^L \psi_p(y)\chi_i(y)dy = -\sum_{n=1}^{N_1}\frac{b_n}{\sqrt{k_n}}\int_0^L \psi_p(y)\chi_n(y)dy$$
$$-\sum_{n=N_1+1}^{N}\frac{b_n}{\sqrt{k_n}}\int_0^L \psi_p(y)\chi_n(y)dy + \frac{c_p}{\sqrt{\nu_p}}, \quad p=1,M \tag{1}$$

$$\sqrt{k_i}\delta_{pi} = \sqrt{k_p}\,b_p + \sum_{m=1}^{M}\sqrt{\nu_m}\,c_m\int_0^L \chi_p(y)\psi_m(y)dy, \quad p=1,N \tag{2}$$

where k_i is the wavevector of the impinging mode, δ_{pm} is the Kronecker delta, $\psi_n(y)$ are the transverse modes in the two parallel channels, $\chi_m(y)$ are the transverse modes in the main lead, N_1 is the number of transverse modes in the upper channel, N is the total number of transverse modes

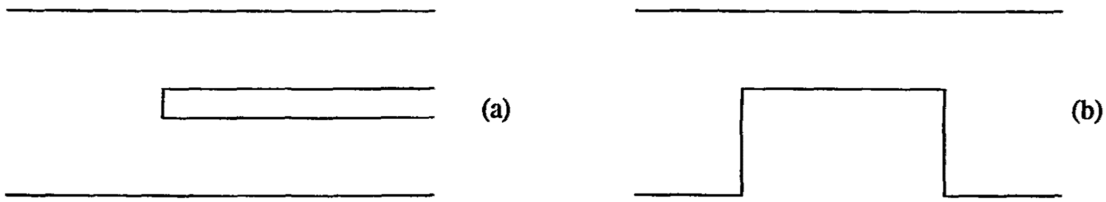

Fig. 1 (a) Waveguide which bifurcates into two parallel channels, (b) Waveguide with constriction of finite length.

in the parallel channels and M is the number of transverse modes in the main lead. The amplitudes of the modes propagating to the right are denoted by b_n, the ones of the modes propagating to the left by c_n.

A principal quantity for our problem is given by the transmission coefficient from one of the two parallel channels into the other. The results are shown in Fig. 2(a), where the energy is expressed in units of the threshold for propagation of the first mode in the main lead. The solid curves represent the square moduli of the transmission coefficients for a waveguide of width L which splits into two channels of width 0.5 L (therefore separated by an infinitely thin wall); the dashed curves are instead for channels 0.45 L wide and separated by a distance 0.1 L. Only T_{ij} with $i = j$ have been plotted, because the off-diagonal terms are vanishingly small. Each transmission coefficient has non zero values only for energies close to the threshold for propagation of the corresponding mode. It reaches a maximum at the threshold or slightly above it; then decays rapidly. Except for energies extremely close to the thresholds, these coefficients are rather small, so that the amplitude of the wave function at the interface with the channel which is not "driven" (into which there is no injection) will be negligible.

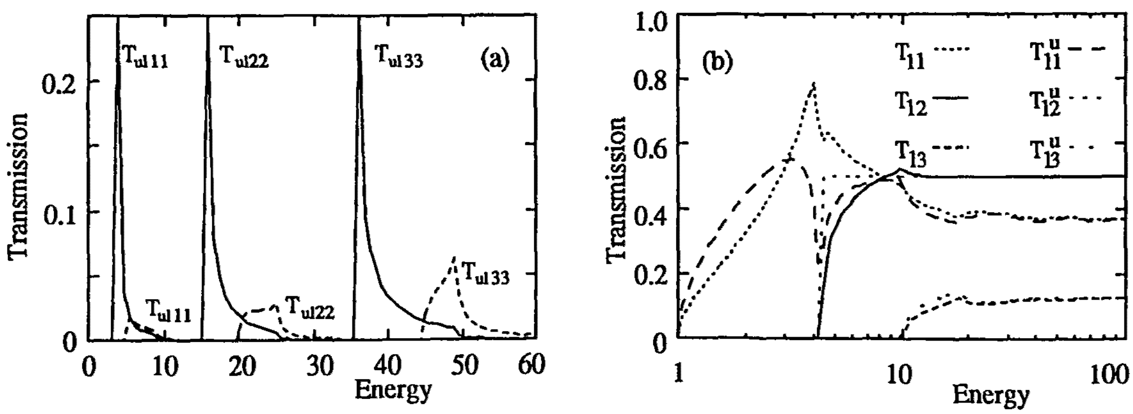

Fig. 2 (a) Transmission coefficients from the lower channel into the upper channel for an electron waveguide which splits into two parallel channels at an interface. (b) Transmission coefficients to the wider lead for impinging with mode 1 from one of the parallel channels. Curves labelled with T^u_{1j} are with both channels present, the others with the upper channel closed by a hard wall. The energy is expressed in units of the threshold for propagation of the lowest mode in the wider lead.

Therefore we expect that the addition of a hard wall, closing the entrance of this last channel and forcing the wave function exactly to zero, should not perturb the transmission coefficients from the "driven" channel to the main lead. This fact has been verified numerically and the results are shown in Fig. 2(b) for parallel channels of width 0.5 L. The solid curves labelled with T_{1j} represent the transmission from mode 1 into mode j, when only one channel is present, the ones labelled with T^u_{1j} are, instead, for the situation with both channels open. The threshold for propagation of the first mode in the parallel channel is at energy $\varepsilon = 4$. We notice that three different regions with different behavior can be identified: below threshold the coefficients, which represent the amplitude of evanescent modes, differ significantly, just above threshold the difference is still present, but tends to die out, until, beyond $\varepsilon = 20$, the coincidence is practically exact, as expected from the behavior of the coupling coefficients. For wider separations of the channels the coincidence is reached closer to the threshold. Analogous results can be obtained for the transmission from modes different from the first.

3. Single constriction and parallel channels

Consider now a structure with a single constriction of finite length (Fig. 1(b)), the vectors **f** and **g** of the amplitudes of the right and left propagating modes can be expressed by the scattering expansions

$$\begin{aligned} \mathbf{f} = \lim_{n\to\infty} \mathbf{Ta} + (\mathbf{R\Gamma R\Gamma})\mathbf{Ta} + (\mathbf{R\Gamma R\Gamma})(\mathbf{R\Gamma R\Gamma})\mathbf{Ta} + \cdots \\ + (\mathbf{R\Gamma R\Gamma})^n\mathbf{Ta} = (\mathbf{I} - \mathbf{R\Gamma R\Gamma})^{-1}\mathbf{Ta} \end{aligned} \tag{3}$$

$$\begin{aligned} \mathbf{g} = \lim_{n\to\infty} \mathbf{\Gamma R\Gamma Ta} + \mathbf{\Gamma}(\mathbf{R\Gamma R\Gamma})\mathbf{R\Gamma Ta} + \cdots \\ + \mathbf{\Gamma}(\mathbf{R\Gamma R\Gamma})^n\mathbf{R\Gamma Ta} = \mathbf{\Gamma}(\mathbf{I} - \mathbf{R\Gamma R\Gamma})^{-1}\mathbf{R\Gamma Ta}, \end{aligned} \tag{4}$$

where **T** is the matrix of the transmission coefficients for an interface like the one of Fig. 1(a), but with only one channel (the entrance of the other one is closed with a hard wall), from the wide lead to the narrow one, **R** is the matrix of the reflection coefficients for the same interface from one of the parallel channels into itself. **Γ** is a diagonal matrix whose elements represent the phase shift undergone by each mode across the length of the constriction. These expressions are easily obtained considering the amplitudes of the modes which just propagate through the first interface, the amplitudes of the ones which reflect off the second interface and again off the first, and all the higher order contributions.

For energies far enough from threshold all the terms of these expansions become negligible except for the first one in Eq.(3). If also the second channel is present, **T** needs to be replaced by $\mathbf{T}'$, the transmission matrix for the condition with both channels open, and additional terms representing the coupling between the channels must be added as, for example, $\mathbf{R}'\mathbf{\Gamma T}_{ul}\mathbf{\Gamma T}'\mathbf{a}$, where $\mathbf{T}_{ul}$ is the transmission matrix from one channel to the other. However, since $\mathbf{T} \approx \mathbf{T}'$, and $\mathbf{T}_{ul}$ vanishes far from the thresholds, $\mathbf{f} = \mathbf{Ta}$, as with only one channel. This means that also the probability current is unchanged. The same is true for both channels: the probability current in each channel is unaffected by the existence of the other one. The total probability current beyond the bifurcation is therefore the sum of the ones associated with each channel, since the total probability current has to be conserved along the structure. This directly implies the additivity of conductances, since conductance is the sum of the current probabilities obtained injecting one mode at a time. More complicated considerations are needed to examine the situation near the thresholds and will be discussed elsewhere [6].

Conductance additivity is thus the consequence of the decoupling property of the interfaces: we expect it to hold for any shape and length of the channels. We have verified this conjecture by numerically evaluating the conductance for many structures. The results for one of these structures are shown in Fig. 3(a), where the solid line represents the conductance for the situation with both channels open, and the dashed line represents the sum of the conductances for each channel. There is a sketch of the structure in the inset of the figure. The correspondence between the two curves is almost exact, except for small discrepancies around the thresholds, where the coupling between the two channels is not negligible.

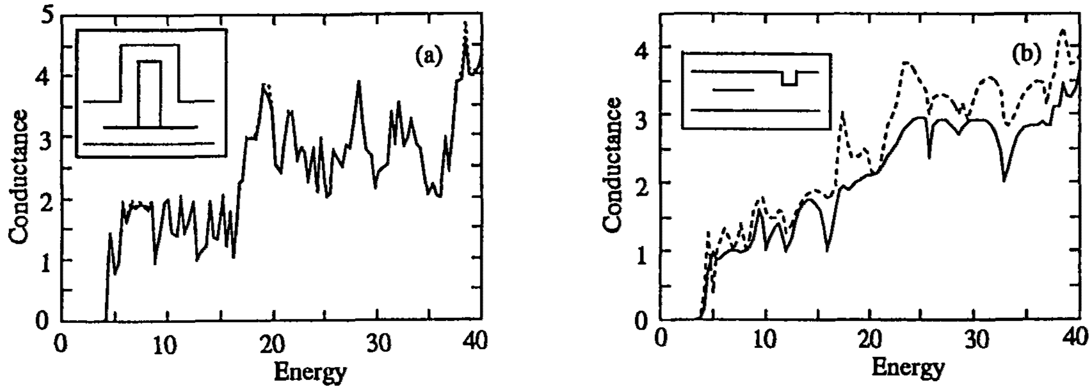

Fig. 3 Conductance (in units of $2e^2/h$) versus energy for (a) two parallel channels of different length, and (b) for two parallel constrictions followed by an additional scattering center. The energy is expressed in units of the threshold for propagation of the lowest mode in the widest section.

We have shown therefore that additivity breaks down if the geometry is modified in such a way as to introduce some significant coupling between the channels. One possible way of doing this is by placing an additional scattering center outside the double channel region, which will cause some reflection of the output of each channel into the other one. Figure 3(b) shows the results for a structure with a constriction following the split section. In this case the probability flux in each channel is perturbed by what flows into the other and conductances do not add.

4. Conclusions

We have investigated the reasons for the classical addition of the conductances of parallel mesoscopic channels. We have determined that it is a consequence of the lack of transmission between the channels and of the conservation of the total probability current through a structure without particle sources or sinks. The sum law for conductances holds for any geometry of the channels into which the electron waveguide divides, as long as the region outside the interfaces does not contain any other scattering center.

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