ELECTRON-PHONON SCATTERING IN QUANTUM WIRES SUBJECTED TO HIGH MAGNETIC FIELDS

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Abstract

We present numerical calculation of the electron-longitudinal optical phonon and electron-surface optical phonon scattering rates in a GaAs quantum wire subjected to high magnetic fields. The matrix elements for the scattering rates are calculated by assuming confined modes for the phonons and the exact wavefunctions for the electrons which are found by solving the Schrödinger equation in a magnetic field. We find that a magnetic field increases both the LO and SO phonon scattering rates by increasing the overlap between the electronic wavefunctions and the phonon modes. This will cause the mobility to decrease with increasing magnetic field thereby giving rise to a positive magnetoresistance. Our calculated scattering rates will be important for performing Monte Carlo simulation of high field magnetotransport in quantum wires and also for studying magneto-phonon effects.

Hot electron transport in quasi one-dimensional systems have been studied by a number of researchers in the past [1]. Most of their models treat the electron confinement correctly, but neglect confinement effects on the phonons. While this approach does not make a significant difference in most cases, there is one case where it fails miserably. That case corresponds to the situation when a *magnetic field is present.* A magnetic field changes the overlap between *confined* phonon fields and electron wavefunctions by skewing the wavefunctions away from the peak of the confined phonon mode. As a result, the scattering rate (and hence the mobility of electrons) will depend significantly on the magnetic field. This is an effect that could not be modeled or studied without accounting for the confined nature of phonon modes.

In this paper, we have studied the effect of a magnetic field on the electron-longitudinal optical phonon and electron-surface optical phonon scattering rates in a GaAs quantum wire. In calculating the scattering rates, we followed the model of Stroscio [2] and Kim. et. al. [3], who have used a macroscopic model for confined phonon modes in a quantum wire and a generalized Fröhlich interaction. We basically use their prescription, but with two modifications: we replace their electron wavefunctions (sine functions) with the exact electron wavefunctions under a magnetic field, and also we use the exact density of hybrid magnetoelelctric states instead of the usual one-dimensional density of states. The electron wavefunctions and the density of magnetoelectric states are found by solving the Schrödinger equation in a quantum wire subjected to a magnetic field. The solution procedure employs a numerical finite difference scheme. This method has been described in Ref. 4.

In our model, the LO phonon scattering rate is given by

$$\frac{1}{\tau_{LO}(E,n)} = \sum_{p'=1}^{pmax} \frac{e^2}{4\pi\epsilon_0}\omega_{LO}\left(N+\frac{1}{2}\mp\frac{1}{2}\right) I_{LO} DOS(E',p') \ , \tag{1}$$

where the upper sign refers to absorption and the lower to emission, In the expressions, E and n are in the initial electron energy and magnetoelectric subband index, E' and p' are the final electron energy and subband index, ω_{LO} is the LO phonon frequency, N is the phonon occupation factor, DOS is the density of magnetoelectric states as a function of E' and p', and I_{LO} is the integral

$$I_{LO} = \left(\frac{1}{\epsilon_r(\infty)} - \frac{1}{\epsilon_r(0)}\right) \frac{(2\pi)^2}{L_y L_z} \sum_{m'=1,3,5,...} \sum_{n'=1,3,5,...} \left\{ \frac{4 P_{m'n'}}{\left[k_x^2 + \left(\frac{m'\pi}{L_y}\right)^2 + \left(\frac{n'\pi}{L_z}\right)^2\right]^{\frac{1}{2}}} \right\}^2 , \quad (2)$$

where $\epsilon_r(0)$ and $\epsilon_r(\infty)$ are the low- and high-frequency relative permittivities, L_y and L_z are the transverse widths of the quantum wire, k_x is the wavevector along the length of the wire and $P_{m'n'}$ is the overlap integral

$$P_{m'n'} = \int_{-L_y/2}^{L_y/2} \int_{-L_z/2}^{L_z/2} \frac{dy}{L_y/2} \frac{dz}{L_z/2} \phi_{in}(y)\phi_{fin}(y)\psi_{in}(z)\psi_{fin}(z) cos\left(\frac{m'\pi}{L_y} y\right) cos\left(\frac{n'\pi}{L_z} z\right) , \quad (3)$$

where $\phi_{in}(y)$ ($\psi_{in}(z)$) and $\phi_{fin}(y)$ ($\psi_{fin}(z)$) are the y- (z-) components of the wavefunctions of the initial and final states in a magnetic field.

The SO phonon rates are given by

$$\frac{1}{\tau_{SO}(E,n)} = \sum_{p'=1}^{pmax} \frac{e^2}{4\pi\epsilon_0} \omega_{SO} \left(N + \frac{1}{2} \mp \frac{1}{2}\right) I_{SO} DOS(E', p') , \quad (4)$$

where

$$I_{SO} = \left(\frac{2\pi C''}{\omega_{SO}} P_s\right)^2 , \quad (5)$$

and C' is the amplitude of the surface optical phonon mode as defined in Ref. 3. Also

$$P_s = \frac{1}{cosh(\alpha L_y/2) cosh(\beta L_z/2)} \int_{-L_y/2}^{L_y/2} \int_{-L_z/2}^{L_z/2} \frac{dy}{L_y/2} \frac{dz}{L_z/2} \phi^2(y)\psi^2(z) cosh(\alpha y) cosh(\beta z) \quad (6)$$

In the above expressions, the only approximations that we made are the following: we have neglected any effect of the magnetic field on the phonons and we have also neglected hot phonon effects in assuming the Bose-Einstein factor for the phonon occupation numbers.

The test system for which we present results is a GaAs wire whose widths along the y- and z-directions are 500 Å and 40 Å respectively. Only one subband is occupied in the z-direction for even the highest electron energy that we consider. However, multiple magnetoelectric subbands are occupied in the y-direction for the highest electron energies. In Fig. 1, we show the SO and LO phonon emission rates as a function of electron energy in the lowest subband for various values of the magnetic field and in Fig. 2, we show the same rates as a function of magnetic flux density for a fixed electron energy. For the SO phonon, only the symmetric mode was considered. Note that both the LO and the SO phonon scattering rates increase with increasing magnetic field because of an increase in the overlap integrals $P_{m'n'}$ and P_s. The integral $P_{m'n'}$ increases since the magnetic field breaks the orthogonality of the electron wavefunction in one mode and the phonon amplitude in another mode, whereas the integral P_s increases since the magnetic field skews the wavefunction towards the wire edges where the SO phonon field peaks.

In conclusion, we have found that a magnetic field increases the overall phonon scattering in a quantum wire somewhat leading to a positive magnetoresistance. Our results for the scattering rates are very useful in the study of hot electron magnetotransport in one-dimensional systems.

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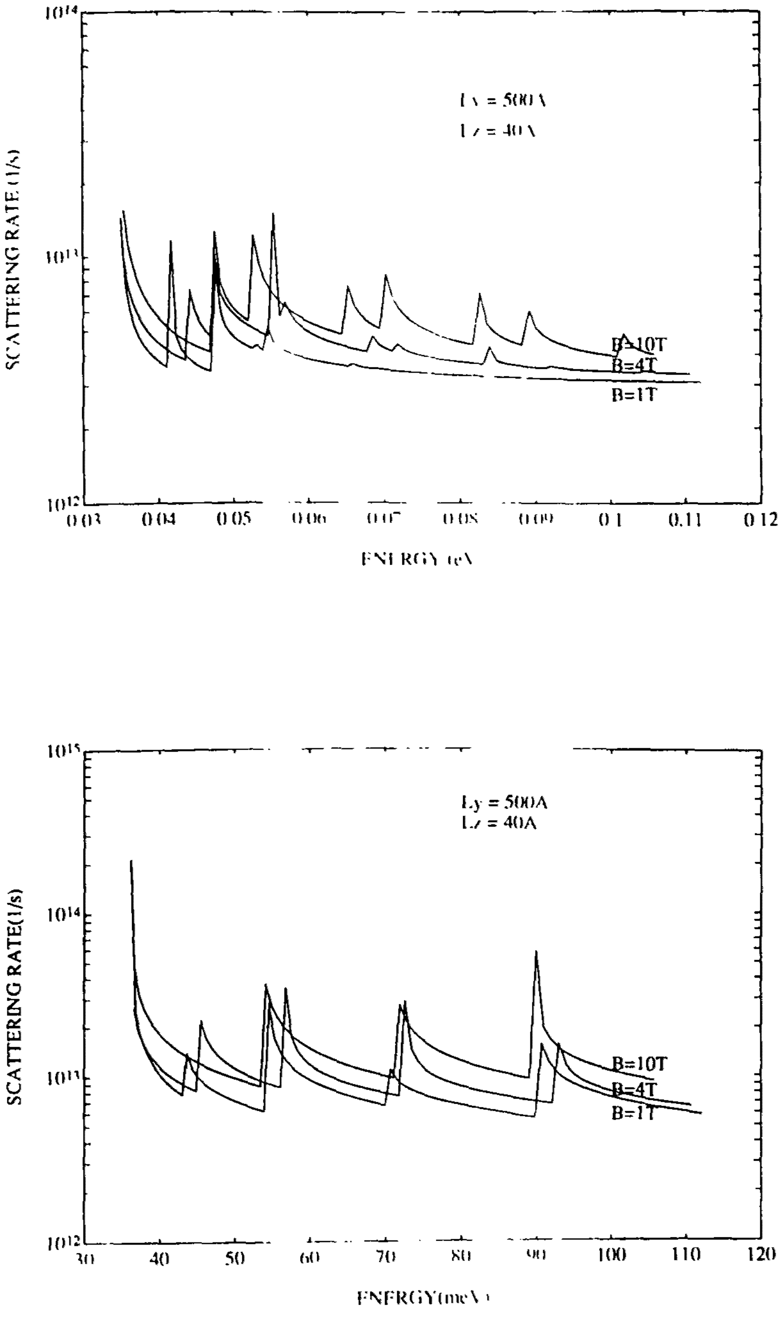

Figure 1: The SO and LO phonon emission scattering rates as a function of energy in a GaAs quantum wire of cross-section 500 Å× 40 Å. The results are plotted for various values of the magnetic flux density. The top figure is for SO phonons and the bottom figure for LO phonons.

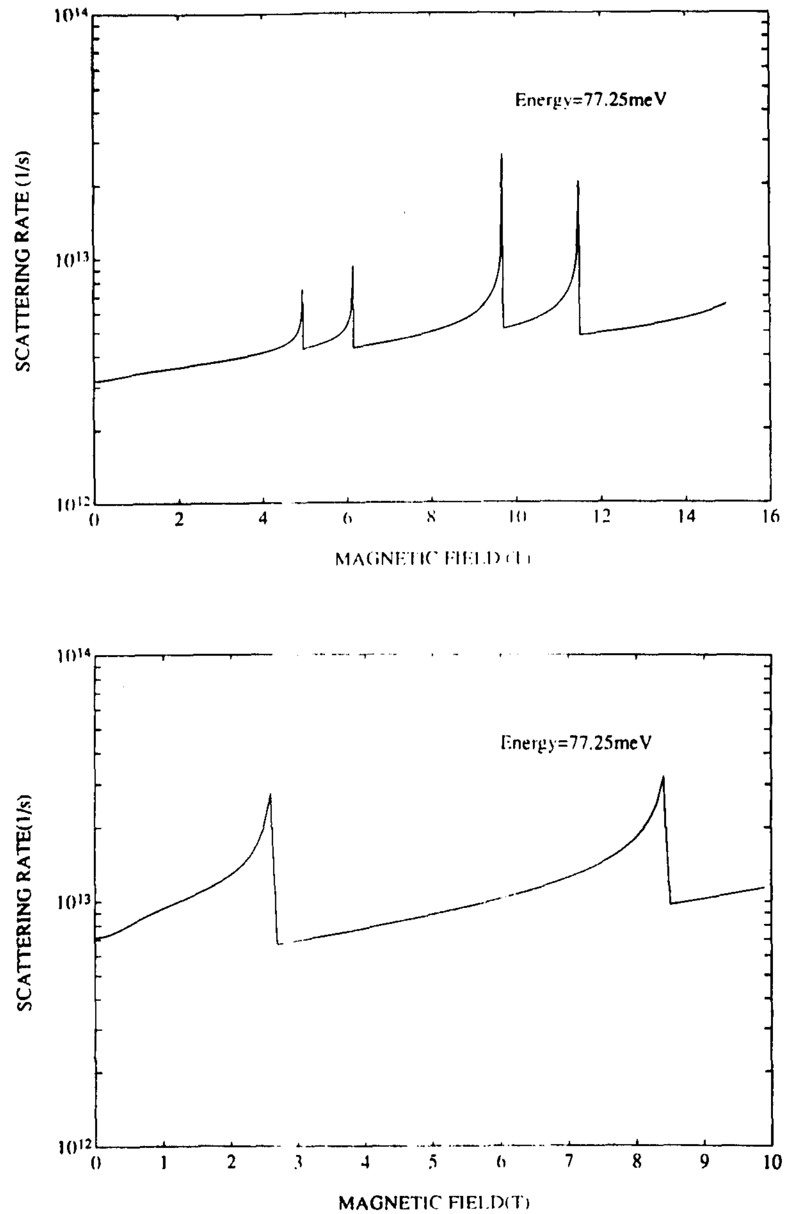

Figure 2: The SO and LO phonon emission scattering rates as a function of magnetic flux density in a GaAs quantum wire of cross-section 500 Å× 40 Å. The top figure is for SO phonons and the bottom figure for LO phonons.

REFERENCES

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