EXTENSION OF A NEW MOMENT METHOD TO FIELD-RETARDED BALLISTIC TRANSPORT IN SEMICONDUCTOR DEVICE STRUCTURES

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Abstract

Ballistic effects play a central role in understanding transport in submicron devices. We present a new moment method for ballistic transport in a one-dimensional semiconductor device structure. In the ballistic regime, our moment method is computationally efficient compared with spectral and Monte Carlo methods. Here, we have extended this moment method from field-assisted to field-retarded flow. Results for the spatial, field, and (weak) collisional dependencies of low-order moment variables are presented.

Summary

In a previous report,[1] a new moment method was developed and applied to the study of field-assisted ballistic transport in a one-dimensional semiconductor device structure. This moment method is based on the characteristic solution of the collisionless Boltzmann equation. We have found the method to be computationally efficient, for a given level of accuracy, compared with conventional Boltzmann solution methods (spectral, direct iterative, or Monte Carlo).

Here, we extend our method to the field-retarded case. The distribution function is parameterized in a form corresponding to the exact collisionless characteristic solution for field-retarded flow. Upon substituting the parameterized distribution into Boltzmann's equation and evaluating velocity moments, a set of nonlinear ordinary differential equations governing the distribution parameters follows. Macroscopic quantities such as carrier concentration and mean velocity are readily calculated.

We present results for a linear electric potential $U(x) = -|qE|x$, $0 \leq x \leq L$, where q is electron charge, E is the electric field, and L is the length of our device structure. (Note, our approach is also valid for general nonlinear potentials.) The characteristic solution[2] for this problem suggests the distribution parameterization $f(x,v) = g(x)\exp(-\beta v^2)$ for $v < \hat{v}(x)$, and $f(x,v) = 0$ for $v > \hat{v}(x)$; where $\beta = m^*/2k_BT$, m^* is an appropriate effective mass, k_B is Boltzmann's constant, and T is absolute temperature. The distribution f is parameterized by two functions g and $\hat{v}$ depending on position x. We have assumed the injection of an equilibrium Maxwellian distribution of carriers at $x = L$, while no carriers are injected at $x = 0$.

Weak scattering is included in our formulation by means of a relaxation time τ. The zeroth- and first-order velocity moments yield

$$\frac{\mathrm{d}}{\mathrm{d}x}[g\exp(-\beta\hat{v}^2)] = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\hat{v}^2}{2}\right) = -\frac{1}{m^*}\frac{\mathrm{d}U}{\mathrm{d}x} + \frac{v_{\mathrm{th}}}{\sqrt{\beta\pi}\lambda}\left[\frac{\exp(-\beta\hat{v}^2)}{1+\mathrm{erf}(\sqrt{\beta}\hat{v})}\right].$$

Above, v_{th} is the thermal velocity $(2k_BT/m^*)^{1/2}$ and λ is the mean free path $v_{\mathrm{th}}\tau$. We impose the boundary conditions $\hat{v}(0) = 0$ and $g(L) = 1.0$ (dimensionless units). Our natural parametric representation substantially improves computational efficiency in that the phase-space Boltzmann equation has been reduced to two *ordinary* differential equations in *real* space.

To aid in the presentation of results, we introduce the dimensionless quantities $\bar{\lambda} = \lambda/L$ and $R = |qE|L/k_BT$. The smallest value of λ used is $0.6L$, which keeps the analysis in the ballistic regime. Figure 1 portrays the carrier concentration n in terms of dimensionless density $\sqrt{\pi}n(x/L)$ (normalized to the injected distribution) versus position. For a given λ, the concentration is a maximum at $x = L$ and is a minimum at $x = 0$. At $x = L$, concentration is smallest for $\lambda = \infty$, increasing as $\bar{\lambda}$ decreases. This behavior is consistent with the fact that increasing scattering strength confines a greater number of carriers at the low end of the potential. Complementary behavior is observed at $x = 0$.

In Fig. 2, the logarithm of current density J is plotted as a function of R for $\lambda = 5.0$. We find excellent agreement with the expected log-linear result from thermionic-emission theory.

Currently, we are extending our moment method in three ways: (1) a unified description of collision-dominated and collisionless transport, (2) generalization to two and three dimensions, and (3) application to quantum transport by means of the Wigner-Boltzmann equation.

References

[1] S. C. Tiersten and Y. L. Le Coz, "A New Moment Method for Ballistic Solution of Boltzmann's Equation in Semiconductor Device Structures", *Proceeding of the ISDRS-91*, Charlottesville, Va., pp. 101-4.

[2] Y. L. Le Coz, *Semiconductor Device Simulation: A Spectral Method for Solution of the Boltzmann Transport Equation*, Ph. D. thesis, Massachusetts Institute of Technology, 1988.

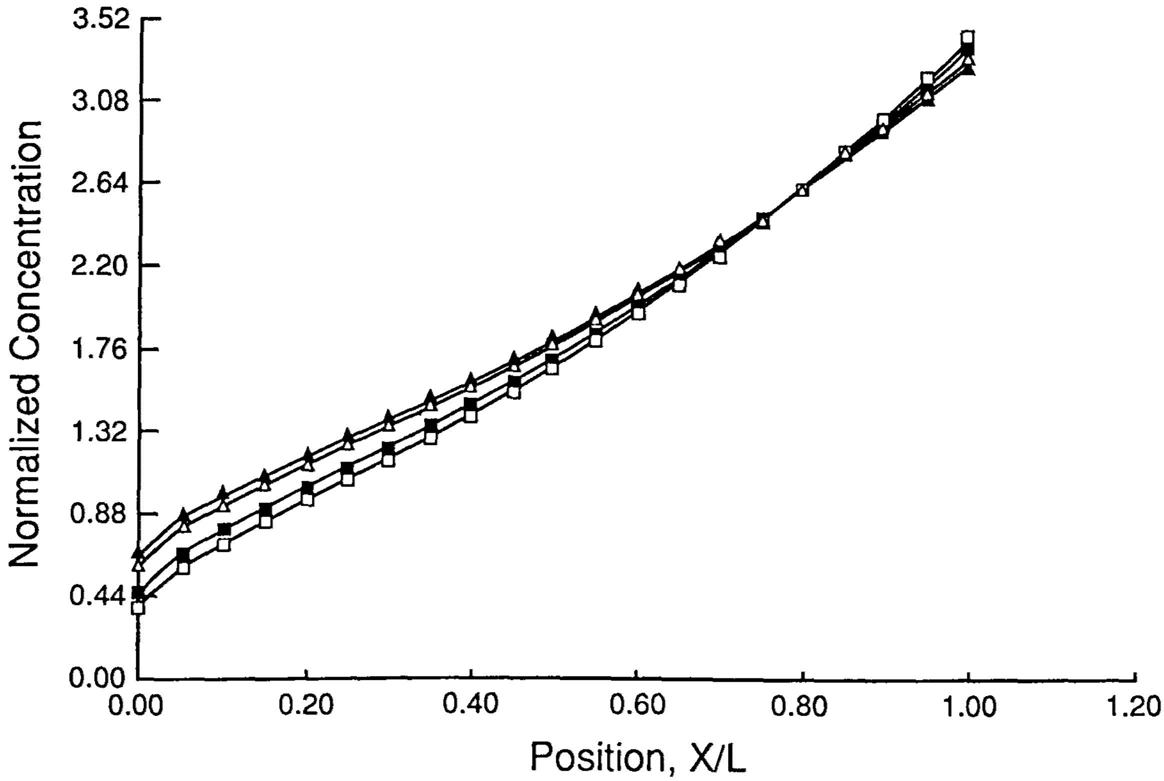

Figure 1: Carrier concentration for field-retarded flow for R = 1.0. Solid triangles $\bar{\lambda} = \infty$, open triangles $\bar{\lambda} = 5.0$, solid squares $\bar{\lambda} = 1.0$, and open squares $\bar{\lambda} = 0.6$.

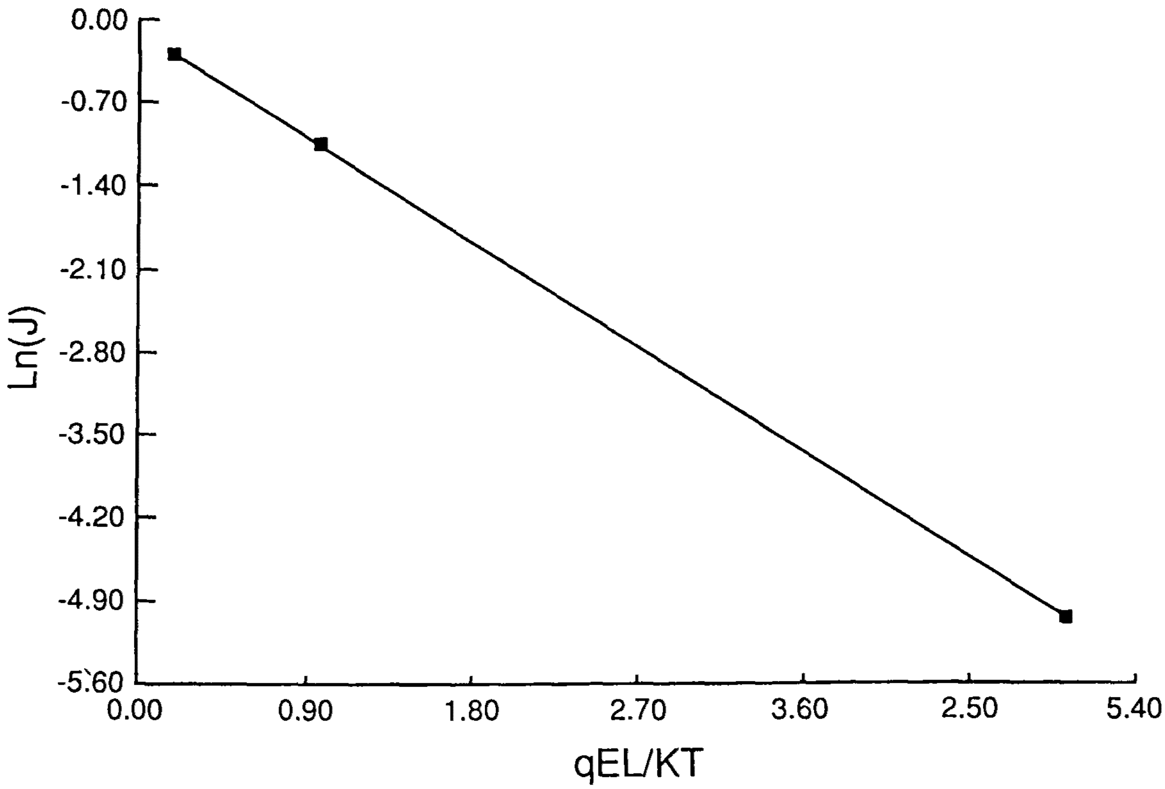

Figure 2: Ln(J) vs. R = |qE| L/K_BT for field-retarded flow, $\bar{\lambda} = 5.0$.