# Algebraic Multigrid Poisson Equation Solver

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## Abstract

This paper discusses highly efficient and robust Multigrid numerical solver for the solution of the Poisson equation in device simulators. Fast and accurate solver is needed when 3D device or multi-scale simulations are performed. It is important to note that the algorithm complexity of commonly used iterative methods, such as Successive-Over-Relaxation (SOR) and Conjugate Gradients (CG), is typically higher, which means time cost of the numerical solver will become predominant with increasing number of mesh points.

## Introduction

The complexity of SOR is $O(N^{1.5})$, where N is the number of total grid points. Compared to this, the Multigrid complexity is on the order of O(N) [1]. Unlike conventional single level iterative methods, Multigrid method uses coarse mesh. The original mesh is treated as the finest mesh. The coarser grid is a subset of a fine grid. Either computing residuals or approximate solutions are transferred back and forth between each grid level depending on the type of Multigrid. Figure 1 schematically shows how standard Multigrid works in a 5-level V cycle.

## Multigrid and Its Variants

There are three stages in a complete Multigrid solver (as shown in Figure 2). Information about coarser levels is obtained either manually or automatically in the setup stage – stage 1. The second stage is optional. This involves preprocessing of the initial guess, which is strongly recommended for nonlinear problems, because the capture zone of a nonlinear problem is small, i.e. stability of the nonlinear problem is low. The third stage is the core part of a Multigrid solver, which is iterated until a convergence criterion is met. Standard Multigrid is the most intuitive with the least overheads in computation. However, standard Multigrid requires $2^n+1$ grid points in each direction for aggressive coarsening. It also requires manual setup of coarser grid, which becomes difficult for complex geometries.

To overcome these drawbacks, few variants of the standard Multigrid are implemented and numerically tested [2]. Algebraic Multigrid (AMG) is an alternative in the setup stage, which imposes no requirement for the mesh. Coarser meshes and corresponding coefficients are automatically generated based on the finest mesh requirements. Full Multigrid (FMG) is used to preprocess initial guess for nonlinear problem to improve the robustness of Multigrid solver. Standard Multigrid can only deal with linear system. However, Full Approximation Scheme (FAS) can directly solve nonlinear problem without linearization.

## Application on a 2D MOSFET

Conventional SOR, AMG, and FAS are tested to calculate the equilibrium potential profile of a 2D MOSFET device shown in Figure 3. Figure 4 shows that AMG barely has advantages over SOR because nonlinear Poisson's equation is linearized each iteration cycle, which changes the transient linear system. Therefore, setup is performed each iteration. Figure 6 shows that FAS performs much better than SOR in terms of speed. Figure 5 shows a cutline of converged potential. SOR generates different results with different initial guess, while the results of FAS are independent from initial guess because of FMG as a preprocessor.

The above results suggest that Multigrid method shows large advantages over SOR to benefit 3D or multi-scale device simulations.

## References

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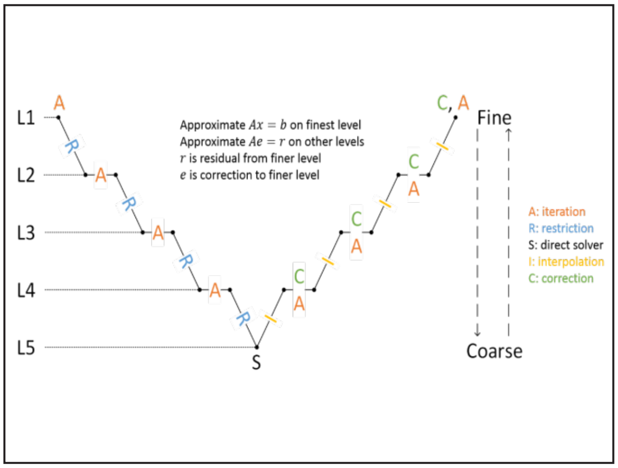


Fig. **1**. Scheme of a Multigrid V Cycle

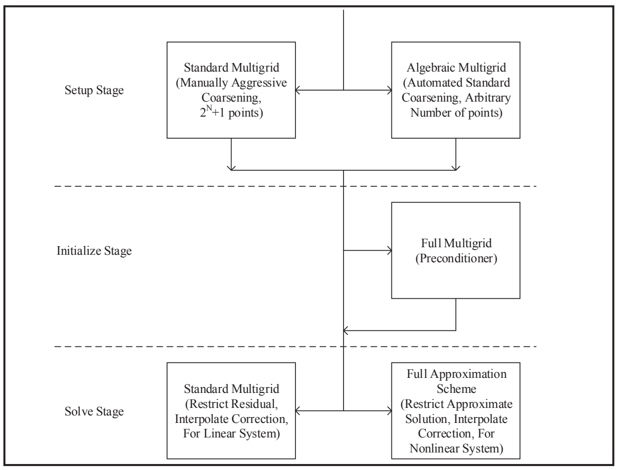


Fig. 2. Stages and Components of a Multigrid Solver

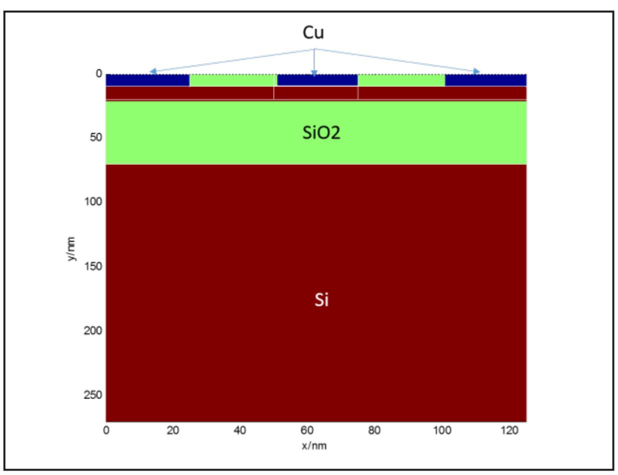


Fig. 3. 2D MOSFET Geometry

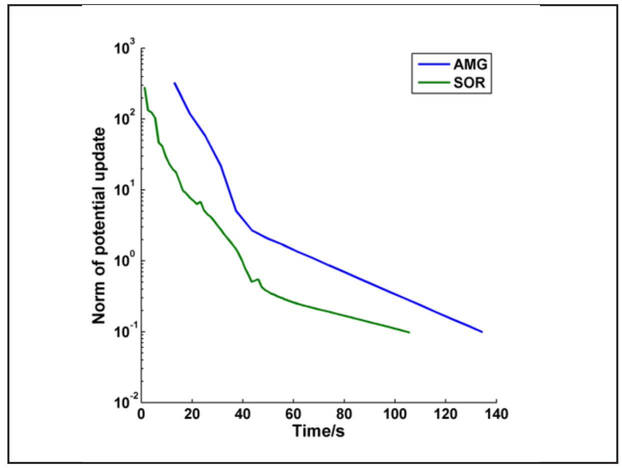


Fig. 4. Convergence of SOR and AMG on 2D MOSFET

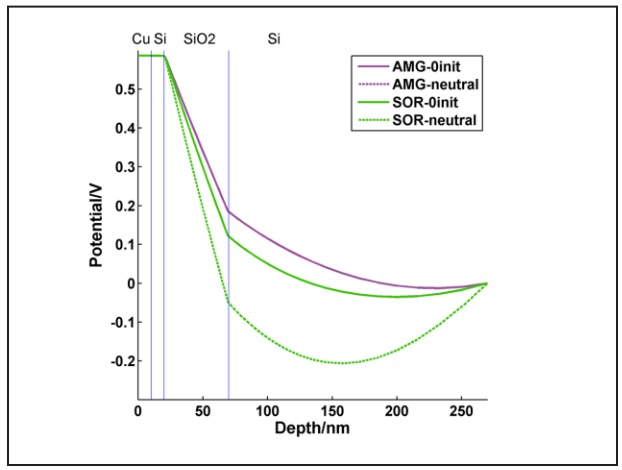


Fig. 5. Vertical Cutline of Potential Profile

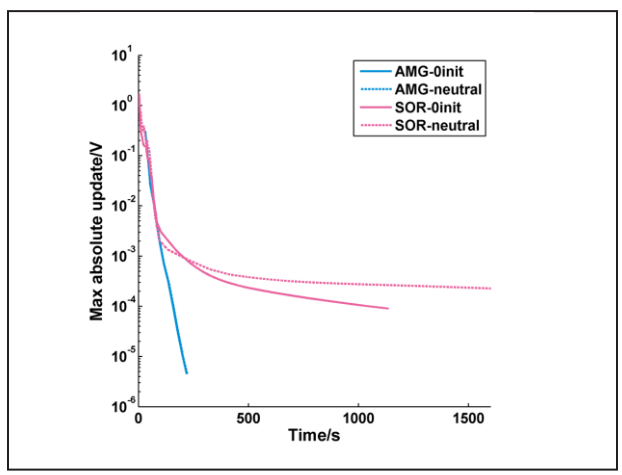


Fig. 6. Convergence of SOR and Hybrid Multigrid on 2D MOSFET