Reformulation of quantum noise: when indistinguishable becomes distinguishable?

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Introduction

The study of noise effects on the performance of quantum devices is difficult either from a computational or experimental point of view. Quite often noise predictions are obtained using DC parameters (like single-particle transmission coefficient) into simple noise expressions, rather than by a direct evaluation of the time-dependent fluctuations. In this conference, we show that an explicit computation of the quantum noise in quite simple scenarios provides surprising and unexpected results.

Reformulation of quantum noise

In a recent work [1], we proved that two (fermions) electrons impinging in a potential barrier can be detected at the same place when the transmitted and reflected components become different. Then, indistinguishable electrons behave as distinguishable. The only requirement is to consider (normalizable) wave packets, instead of the infinite-spatially extended scattering states used in the Landauer-Büttiker formalism (LBF) [2]. In Fig. 1 we schematically show these new possibilities of detecting two electrons at the left c) or at the right d), with probability $\mathcal{P}_{\mathcal{LL}}$ and $\mathcal{P}_{\mathcal{RR}}$ respectively. These new two-particle scattering possibilities must be added to the usual ones considered by the LBF. i.e. Fig.1 a) and b). Some relevant and unexplained experiments [3] and [4] are in agreement with these new probabilities. Depending on the size of the initial wave packet and on its central momentum, $\mathcal{P}_{\mathcal{RR}}$ and $\mathcal{P}_{\mathcal{LL}}$ have two limits: the probabilities associated with the LBF (equal to zero) and the probabilities associated with distinguishable particles (for more details see [1] and [5]). In Fig. 2 we report the non-zero probability $\mathcal{P}_{\mathcal{LL}}$ that provides an increment of the noise level above LBF predictions.

At low frequencies, when the displacement current is neglected, current and noise can be computed from the knowledge of the number N of transmitted particles through the barrier during the time t_d:

$$\langle I(E)\rangle = lim_{t_d\to\infty} q\frac{\langle N\rangle_{t_d}}{t_d}, \qquad (1)$$

$$\langle S(E)\rangle = lim_{t_d\to\infty} 2q^2\frac{\langle N^2\rangle_{t_d}-\langle N\rangle_{t_d}^2}{t_d}, \qquad (2)$$

The probabilities $P(N)$ are computed from the direct solution of the two-particle Schrödinger equation with exchange interaction and summarized in TABLE I. We define $\langle N\rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N)N$ and $\langle N^2\rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N)N^2$. As seen in TABLE I, the new $P(N)$, related to $\mathcal{P}_{\mathcal{LR}}$, $\mathcal{P}_{\mathcal{LL}}$ and $\mathcal{P}_{\mathcal{RR}}$ have a direct impact on the current and noise. The usual LBF expressions are obtained if we consider $\mathcal{P}_{\mathcal{RR}} = \mathcal{P}_{\mathcal{LL}} = 0$.

Example: resonant tunneling diode

We apply expressions (1) and (2) with the TABLE I for a simple resonant tunneling diode (RTD). In Fig. 3 we plot the current and Fano factor for our results, LBF and distinguishable particles. The differences in the Fano factor are explained because the exchange correlation is reduced in our results when indistinguishable electrons behave as distinguishable ones.

We have considered a 1D two terminal quantum wire device where only one or two electrons are simultaneously injected from the reservoirs. At high-temperature, when one-particle or two-particle processes are relevant, the extension to many-particle scattering probabilities with self-consistent potentials can be done straightforwardly but will not modify the present qualitative discussion. The

ISBN 978-0-692-50554-0

typical super-poissonian noise in the negative differential resistance does not appear because we did not consider Coulomb correlations, only exchange correlations [6]. At low temperature, many-particle probabilities tend to recover the LBF results, satisfying the fluctuation-dissipation theorem [7].

Conclusions

When studying scattering events with (normalizable) wave packets, new probabilities $P(N)$ appear, allowing the location of two electrons at the same place. See Fig.1 c) and d). Therefore quantum noise is increased over previous predictions. The importance of these new probabilities for the accurate computation of the quantum noise at high temperatures is discussed by computing the current and the Fano factor in a simple RTD.

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Acknowledgment

This work has been partially supported by the "Ministerio de Ciencia e Innovación" through the Spanish Project TEC2012-31330 and by the Grant agreement no: 604391 of the Flagship initiative "Graphene-Based Revolutions in ICT and Beyond".

b \ a		Injection: Transmitted	Injection: Reflected	No injection
Injection	Transmitted	$\frac{\mathcal{P}_{\mathcal{LR}}}{2} f_a f_b$	$\mathcal{P}_{\mathcal{LL}} f_a f_b$	$T_b(1-f_a)f_b$
		0	-1	-1
	Reflected	$\mathcal{P}_{\mathcal{RR}} f_a f_b$	$\frac{\mathcal{P}_{\mathcal{LR}}}{2} f_a f_b$	$R_b(1-f_a)f_b$
		1	0	0
No injection		$T_a f_a(1-f_b)$	$R_a f_a(1-f_b)$	$(1-f_a)(1-f_b)$
		1	0	0

TABLE I: Probability $P(N)$ (upper) that N (lower) electrons are transmitted from left to right reservoir during the time interval t_d. f_i is the Fermi distribution ($i = a, b$) and T_i and R_i the i-wave packet single particle transmission and reflection.

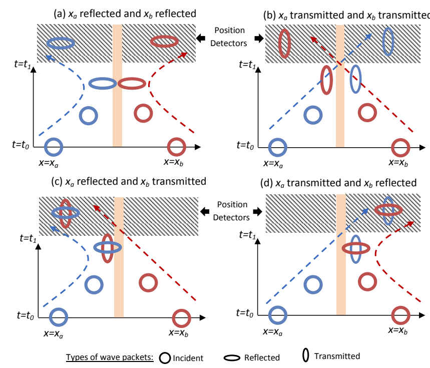

Fig. 1: Scattering of two particles in a potential barrier system. (a) and (b) are the normally accepted final scenarios. (c) and (d) are now also possible final scenarios, both electrons are detected at the same position with probability $\mathcal{P}_{\mathcal{LL}}$ and $\mathcal{P}_{\mathcal{RR}}$.

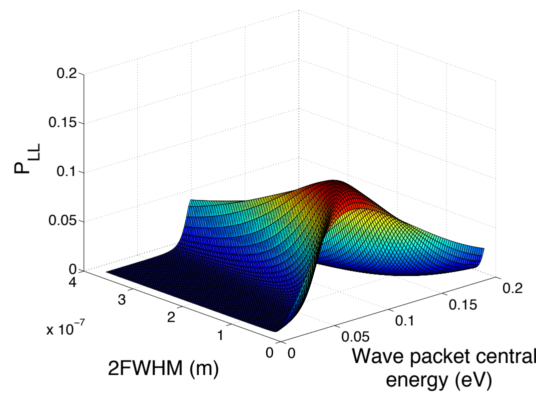

Fig. 2: Probability of finding both electrons (described by a Gaussian wave function) at the left side of a double barrier potential system when there is no applied bias. We see how the zero LBF probability is recovered for large and far from the resonant energy wave packets.

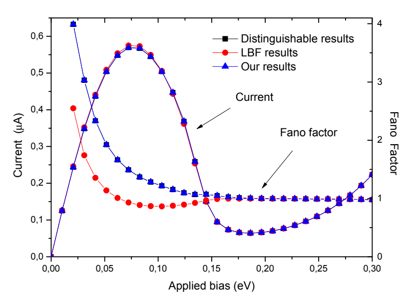

Fig. 3: Current and Fano factor from 3 approaches for a RTD (0.8nm/5.6nm/0.8nm and 0.4 eV height). We set the Fermi level = $0.1eV$ and $T = 300K$.