PiN Diode n-Base Ambipolar Diffusion Equation (ADE): Exponential Solution

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ABSTRACT

The exponential solution of the ADE is presented here. The solution forms a part of PiN diode compact model reduced complexity model dedicated to circuit simulation involving multiple devices and switching sequences.

INTRODUCTION

When the excess carrier concentration exceeds the PiN base doping level by several orders of magnitude (*plasma* condition), the $n \approx p$ is valid (n and p represent the excess electron and hole concentrations) and carrier transport is determined by the ambipolar diffusion equation [1]:

$$D\frac{\partial^2 p(x,t)}{\partial x^2} = \frac{p(x,t)}{\tau} + \frac{\partial p(x,t)}{\partial t} \qquad (1)$$

In the eq. (1) D represents the ambipolar diffusion constant and τ represents the ambipolar carrier lifetime.

ADE EXPONENTIAL SOLUTION

To solve eq. (1) and model plasma carrier distribution, a set of exponential shape functions is used. In steady state, the plasma carrier concentration has a distribution of catenary form with just two exponential basis functions giving:

$$p = Ae^{x/L} + Be^{-x/L} \qquad (2)$$

where L is the diffusion length and A and B are the arbitrary constants determined by boundary conditions at the p+n- and n+n- junctions. In transient operation, more complex profiles are approximated using a number of exponential basis functions with a range of decay length parameters. Defining a normalised position variable $u=x/w$, where w is the base width, the basis functions used in the model are defined in a row vector:

$$N(u) = \lfloor 1 \ \ e^{\alpha_1(u-1)} \ \ e^{-\alpha_1 u} \ \ e^{\alpha_2(u-1)} \ \ e^{-\alpha_2 u} \ \ etc. \rfloor \qquad (3)$$

where, $\alpha_1 = w/L < \alpha_2 < \alpha_3$ *etc*. The dynamics of the plasma are approximated by application of the Rayleigh-Ritz method to eq. (1). Firstly the plasma distribution is approximated by $p= N(u)c(t)$, where $c(t)$ is a column vector of parameters independent of position to be numerically computed as a function of time. During discretisation procedure the solution of the ADE reduces to solving with respect to time a coupled set of ordinary differential equations (ODEs) in $c(t)$ parameters. Discretisation proceeds substituting for p in equation (1), multiplying by a set of linearly independent test functions $N^T(u)$ and integrating between $u_l=x_l/w$ and $u_r= x_r/w$ leads to a set of coupled ODEs describing the dynamics of the plasma parameters $c(t)$. Integrating the second order derivative term by parts and re-arranging gives:

$$\frac{D}{w}\mathbf{Ac} + w\mathbf{B}\left(\frac{\mathbf{c}}{\tau} + \frac{d\mathbf{c}}{dt}\right) = D\left[N^T(u_r)\frac{\partial p}{\partial x}\bigg|_{x_r} - N^T(u_l)\frac{\partial p}{\partial x}\bigg|_{x_l}\right] \qquad (4)$$

$$\mathbf{A} = \int_{u_l}^{u_r} \frac{dN^T}{du}\frac{dN}{du}du \ , \ \mathbf{B} = \int_{u_l}^{u_r} N^T N du$$

Letting the RHS become the known function $f(I_P)$ and using the backward difference time stepping method [2], an expression is obtained for **c**. Since the basis functions are simple, the integrals are easily performed analytically. Hence, parameters $\mathbf{c}(t)$ are obtained in terms of the plasma boundary locations x_l and x_r and the plasma current I_P.

CONCLUSION

An exponential solution of the ADE describing carrier transport through n-Base of the PiN diode has been described.

REFERENCES

[1] A.R. Hefner, *A Dynamic Electro-Thermal Model for the IGBT*, IEEE Trans. on Ind. App. **30**, 394 (1994).

[2] J.D. Lambert, *Numerical Methods for Ordinary Differential Systems – The Initial Value Problem*, John Wiley & sons, England, 1st edition (1991).

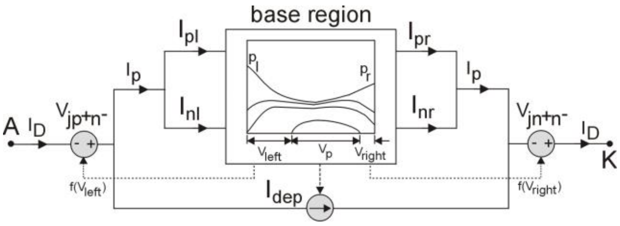

Fig. **1**. A typical PiN diode compact model. In order to describe correctly static and dynamic behaviour of power bipolar devices, it is essential to incorporate into a device model conductivity modulation and non-quasistatic charge storage effects since they are the dominant factors in determining the dynamic and static current and voltage characteristics of these devices. Accurate solution of the ADE plays key part in achieving this.

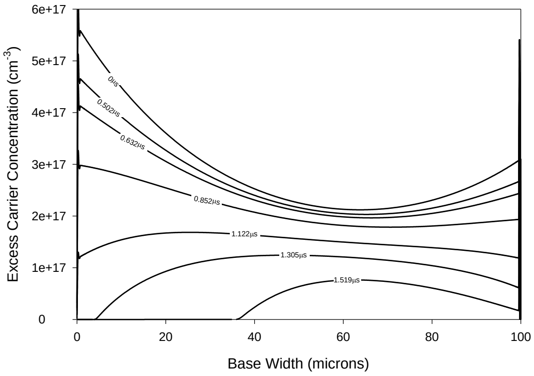

Fig. **2**. Prediction of the PiN n-base plasma evolution - full 2D drift-diffusion model

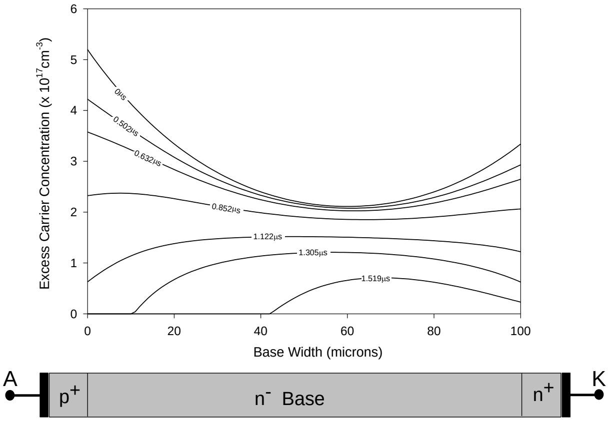

Fig. **3**. Prediction of the PiN n-base plasma evolution – exponential ADE solution. The profiles very similar to ones shown in Fig. 2 are calculated. The simplified PiN diode structure is also shown.