

# Serially-connected Aharonov-Bohm rings with embedded quantum dots

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## INTRODUCTION

Transmission properties of coupled mesoscopic or molecular rings is a topic of interest in computational research [1, 2]. In this paper, multiple nanoscale rings with embedded quantum dots (QDs) are connected serially and investigated using a tight-binding computational model which solves for the transmission through an arbitrary number of rings. The effect of an external magnetic field, via the Aharonov-Bohm (AB) induced phase shift, is included in the system Hamiltonian. Transmission resonance peaks from each QD form into a distinct band structure as the number of rings increases. The AB-effect modifies the band structure and introduces a separate “flux band” for a bi-lateral asymmetry in the QD quasibound state energy values.

## MODEL

The tight-binding approximation to the Schrödinger equation can be written in the following form,

$$-\sum_m V_{n,m} \Psi_m + \varepsilon_n \Psi_n = E \Psi_n \quad (1)$$

A simplified schematic of serially-coupled rings is depicted in Fig. 1 where the coupling parameters and QD site energy values are indicated. The computational model consists of solving a system of coupled linear equations which are built into a matrix equation format for an arbitrary number of rings in series. The resulting transmission is plotted as a function of energy or magnetic flux. Current vs. voltage curves are calculated from the transmission function using a standard formalism based on the scattering theory of transport [3].

## SELECTED RESULTS

Figure 2 shows a contour plot of the transmission for a 10-ring system as a function of

energy (E) and magnetic flux,  $\Phi$ , normalized to the flux quantum,  $\Phi_0 = h/e$ . The inter-QD coupling integrals are set to  $V_1 = 0.1$ , and the couplings between sites in the 1-d leads are all set to  $V_0 = 1.0$ , which we use throughout the discussion as a unit of energy. The electron energy window is  $-2V_0 \leq E \leq 2V_0$ , as set by the tight-binding dispersion relation for the uniform leads. All upper QD site energies are set to  $\varepsilon_u = 0.1$ , all lower sites to  $\varepsilon_d = -0.1$ , and all other sites to  $\varepsilon_m = 0.0$ . Figure 4 shows line plots of the transmission as a function of energy for 3 different flux values. At zero flux, the transmission consists of two distinct QD bands, whose inner edges are located at the QD site energy values. The number of separate resonance peaks in each band is  $n-1$ , where  $n$  is the ring number. For this asymmetric system ( $\varepsilon_u \neq \varepsilon_d$ ), a central flux band appears in between the two QD bands. The flux band results from the phase shift of the AB-effect which spoils the complete destructive interference at  $E=0$ . In Figs. 3 and 5, the current through the 10-ring structure is shown as a function of voltage, for various flux values. It is notable that the application of only a small value of flux can shift the I/V characteristics from semiconducting (with a bandgap) to Ohmic.

Additional features of the serially-coupled, direct-contact ring system, which provides a rich array of resonant features controllable by system parameters, will be presented.

## REFERENCES

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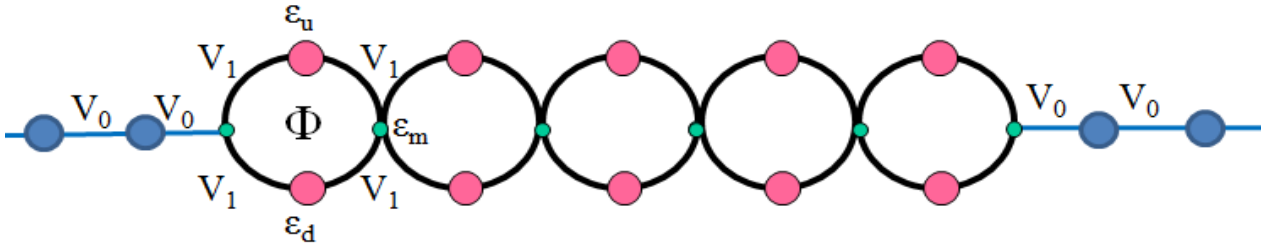


Fig. 1. Schematic of a direct-contact 5-ring system with QDs embedded in the upper and lower arms. The coupling and QD parameters are identical for each ring in series. Parameters:  $\epsilon_u = 0.1$ ,  $\epsilon_d = -0.1$ ,  $\epsilon_m = 0.0$ ,  $V_1 = 0.1$ ,  $V_0 = 1.0$  (same for all Figures).

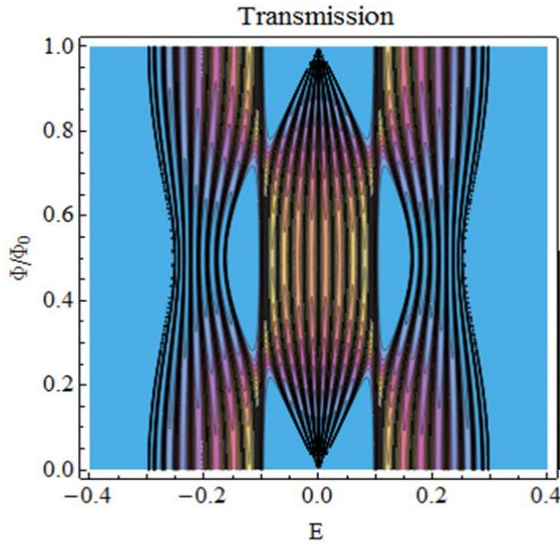


Fig. 2. 10-ring transmission as a function of  $E$  and  $\Phi/\Phi_0$ .

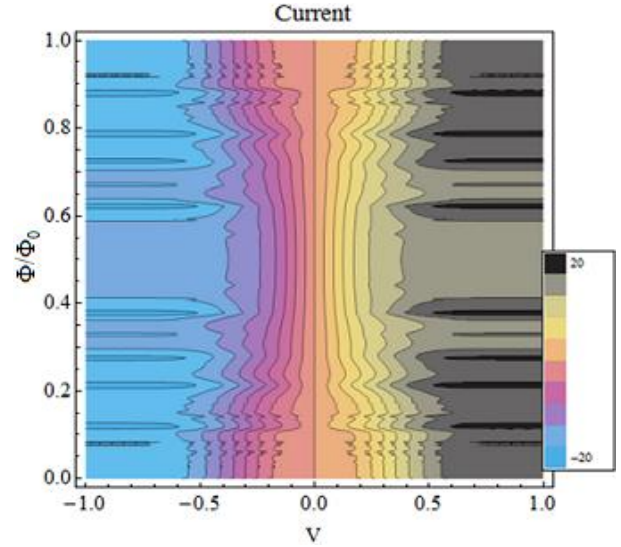


Fig. 3. I/V contours as a function of flux for 10-rings.

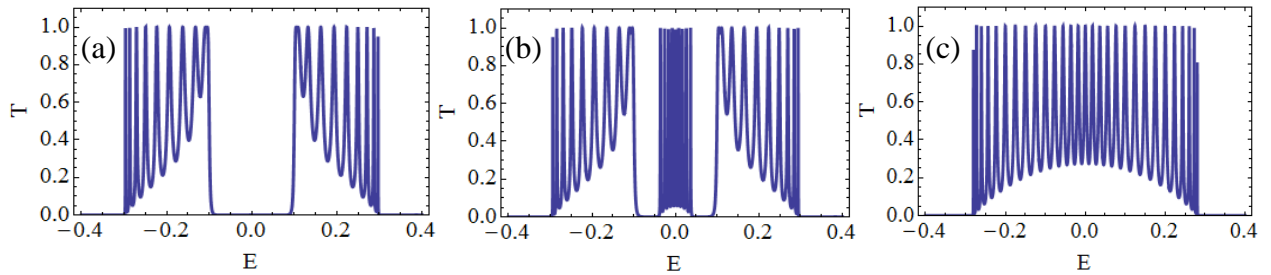


Fig. 4. 10-ring transmission bands for a)  $\Phi/\Phi_0 = 0.0$ , b)  $\Phi/\Phi_0 = 0.1$ , and c)  $\Phi/\Phi_0 = 0.25$ . The central flux band has 9 resonant peaks.

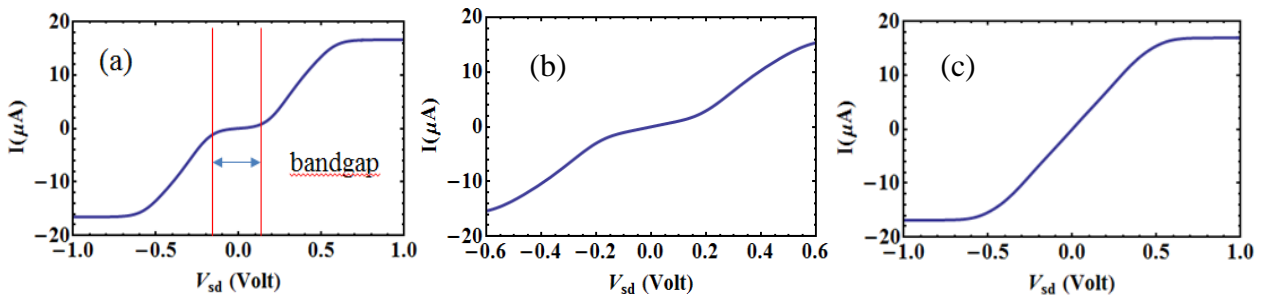


Fig. 5. I/V characteristics of the 10-ring system as a function of magnetic flux: a)  $\Phi/\Phi_0 = 0.0$ , b)  $\Phi/\Phi_0 = 0.1$ , and c)  $\Phi/\Phi_0 = 0.25$ . The system transforms from semiconductive to Ohmic as the flux increases from zero.