Electronic and transport properties of graphene

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Graphene was fabricated first by mechanical exfoliation in 2004 and the conductivity modulation by the gate was demonstrated and the quantum Hall effect was observed in 2005. Since then, various experimental and theoretical investigations have been performed to reveal its exotic properties. Actually, graphene has been a subject of theoretical study prior to the experimental realization because of the peculiar electronic structure also responsible for intriguing properties of carbon nanotubes [1]. Several reviews have already been published [2]–[4]. The purpose of this paper is to give a brief review on characteristic features of electronic states and transport properties in graphene mainly from a theoretical point of view.

The graphene has a honeycomb lattice with two carbon atoms in a unit cell. The Fermi level lies in the so-called π bands consisting of the p_z orbital perpendicular to the plane. The π bands have a linear dispersion and cross at the K and K' points at the corner of the first Brillouin zone. Because the Fermi level lies at the crossing points, electronic properties are dominated by those states. Within the effective-mass approximation or the $\mathbf{k}\cdot\mathbf{p}$ scheme, the electron motion is governed by Weyl's equation for neutrino or the Dirac equation with vanishing mass.

An important feature is the presence of a topological singularity at $\mathbf{k} = 0$. A neutrino has a helicity and its spin is quantized into the direction of the wave vector. The spin eigenfunction changes its sign under a 2π rotation. Correspondingly, the pseudo-spin wave function acquires phase $-\pi$ due to Berry's phase when the wave vector $\mathbf{k}$ is rotated around the origin along a closed contour. For the present pseudo spin, this sign change can be understood in terms of Berry's phase when the wave vector moves along a closed contour around $\mathbf{k} = 0$. The sign change occurs only when the contour encircles $\mathbf{k} = 0$ but not when the contour does not contain $\mathbf{k} = 0$. This topological singularity at $\mathbf{k} = 0$ is the origin of the absence of backscattering in metallic carbon nanotubes [5], [6].

The singularity also leads to the presence of a Landau level at $\varepsilon = 0$ independent of the magnetic-field strength [7]. It is responsible for the singular diamagnetic susceptibility $\chi \propto -\delta(\varepsilon)$ [2], [7], [8]. Further, the nanotube has a strong magnetic anisotropy and tends to align in the field direction [9], which has been extensively used in the observation of the Aharonov-Bohm effect in optical absorption [10]. This singularity is considered as the origin of the peculiar behavior in the transport properties of graphene, such as the minimum conductivity in the absence of a magnetic field, the quantum Hall effect, and the dynamical conductivity [2].

The neutrino equation for $\mathbf{F}$ is invariant under the special time-reversal operation S, $\mathbf{F}^S = K\mathbf{F}^*$, where $\mathbf{F}^*$ stands for the complex conjugate of $\mathbf{F}$ and $K = -i\sigma_y$ satisfying $K^2 = -1$ [1]. When S is operated twice, the wave function changes its sign, i.e., $\mathbf{F}^{S^2} = (\mathbf{F}^S)^S = -\mathbf{F}$. This S symmetry, characteristic to the symplectic universality class, is not destroyed by scatterers unless their potential range is not smaller than the lattice constant. In this case the quantum correction to the conductivity becomes positive and the conductivity exhibits a weak anti-localization behavior [11].

The special time reversal S is different from the real time reversal T in which the K and K' points are exchanged corresponding to the complex conjugate wave functions. When being repeated, the wave function returns to the original with the same sign. A system with this T symmetry belongs to the orthogonal universality class. In the presence of usual scatterers with their potential range larger than the lattice constant, the K and K' points can be regarded as independent because their coupling can safely be neglected. In this case this T symmetry

978-3-901578-26-7

is irrelevant and only the S symmetry prevails. In the presence of short-range scatterers causing inter-valley scattering between the K and K' points, the S symmetry is destroyed and the T symmetry becomes relevant [11], [12]. Therefore, the symmetry crossover between the symplectic and orthogonal classes occurs due to short-range scatterers. The change from the anti-localization behavior in the symplectic case to the weak-localization behavior in the orthogonal case for the quantum correction to the conductivity has been demonstrated [11], [13].

The equi-energy line of graphene in the vicinity of the K or K' points has a small trigonal warping. Effects of the warping can be included as a higher-order $\mathbf{k}\cdot\mathbf{p}$ term [2]. This term destroys the S symmetry and the symmetry of the system changes into unitary when the trigonal warping is appreciable. Further, the lattice strain and the nonzero curvature of the nanotube gives rise to an effective vector potential or Aharonov-Bohm flux, and therefore destroys the S symmetry [2].

Inter-layer interaction in bilayer graphene destroys the linear dispersion into an approximate parabolic dispersion with a trigonal warping [14], [15]. Electronic states of multi-layer graphene depend critically on the number of layers. This becomes clear if we consider only the major coupling terms and neglect other small parameters considered in bulk graphite. In fact, for odd-layer graphene, the Hamiltonian can be decomposed into those of bilayers with different interlayer coupling and that of a monolayer graphene, while for even-layer graphene, the Hamiltonian can be decomposed into those of bilayers only [16]. A perpendiculor electric field or asymmetry between two layers opens up an energy gap in bilayer graphene.

There have been various theoretical and experimental investigations on dominant scattering mechanisms, including effects of charged impurities [17], [18] and environmental dielectric screening effect [19], resonance scattering effects due to strong and short-range scatterers [20], etc. A recent theoretical study showed that the minimum conductivity is sensitive to effective potential range of dominant scatterers [21] in agreement with experiments [22]. The bilayer graphene exhibits the similar behavior [23]. The appearance of effective vector potential due lattice distortion [24] was used for the prediction of magneto-phonon resonances [25]. Interfaces between monolayer and bilayer graphenes were shown to exhibit peculiar dependence on incident angle because of the chiral electron motion, giving rise to valley polarization of transmitted electron [26], and characteristic Landau-level structure [27].

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