Modeling of Current Distribution through Metal-Insulator-Metal Diodes with Tunnel Barrier Roughness

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Since the tunnel current is very sensitive to the tunneling distance, the barrier thickness variability is one of the concerns for integrating the large number of tunnel junctions, e.g., in MRAM [1]. In this study, we numerically investigate the current distribution through metal-insulator-metal diode structures considering the effect of tunnel barrier roughness [2,3], and also discuss the analytical formalism for describing its statistical properties [4] depending on the various parameters.

Figure 1 shows the numerically generated roughness pattern assuming the Gaussian spectral density. We then cut out the samples as shown in Fig. 2, and evaluated the average roughness height $\langle\delta\rangle$ and the average current density $\langle j\rangle$ over each sample area. Figure 3 shows the probability distributions of $\langle\delta\rangle$ plotted as a function of the sample area A. The distributions follow a normal distribution, and the variance $\sigma^2_{\langle\delta\rangle}$ decreases with A due to the statistical averaging effect. Since the spacial correlation is taken into account in the present roughness model, $\sigma^2_{\langle\delta\rangle}$ becomes constant when $A \ll \pi\Lambda^2$ (where Λ is the correlation length), while is inversely proportional to A in the large area limit. As shown in Fig. 4, we have approximated this behavior to the analytical form:

$$\sigma^2_{\langle\delta\rangle} = \Delta^2[1 + A/(\pi\Lambda)^2]^{-1}. \quad (1)$$

Figure 5 shows the probability distributions of $\langle j\rangle$, exhibiting quite distorted form particularly in the small area samples. Since the tunneling current exponentially depends on the barrier thickness, it is lognormally distributed if the thickness is normally distributed. However, with increasing A, the distribution becomes narrower and symmetric in accordance with the central limit theorem.

To analytically describe such behavior, Fenton-Wilkinson approximation [5] was employed. This assumes that when X_i $(i = 1 \ldots N)$ are distributed according to a lognormal distribution: $X_i \sim LN(\mu, \sigma^2)$, then the sum distribution is well approximated by another lognormal distribution: $\sum_i X_i/N \sim LN(\tilde{\mu}, \tilde{\sigma}^2)$, where

$$\tilde{\sigma}^2 = \ln\left(\frac{e^{\sigma^2}}{N} + 1\right), \quad (2)$$

$$\tilde{\mu} = \mu + \frac{\sigma^2 - \tilde{\sigma}^2}{2}, \quad (3)$$

and μ and σ are the mean and the standard deviation of X_i, respectively. Figure 6 shows the numerically simulated distributions of $\langle j\rangle$ together with the results calculated with Eqs. (1)–(3). Note that the analytical approach well describes not only the area dependent peak positions, but also the tail distributions, which are important information for considering, e.g., the scaling limit of the magnetic tunnel junctions used in MRAM.

References

[1] W. S. Zhao et al., *Failure and reliability analusis of STT-MRAM*, Microelectron. Reliab. **52**, 1848 (2012).

[2] V. Serin et al., *TEM and EELS measurement of interface roughness in epitaxial Fe/MgO/Fe magnetic tunnel junctions*, Phys. Rev. B **79**, 144413 (2009).

[3] K. M. Bhutta, *Characterization of MgO barrier by conducting atomic force microscopy*, Microscopy: Science, Technology, Applications and Education **3**, 2022 (2010).

[4] V. Da Costa et al., *Statistical properties of currents flowing through tunnel junctions*, J. Magn. Magn. Mater. **258-259**, 90 (2003).

[5] L. Fenton, *The sum of log-normal probability distributions in scatter transmission systems*, IRE Trans. Comm. Sys., **8**, 57 (1960).

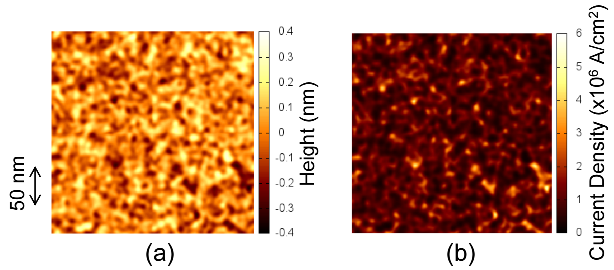

Fig. 1. Roughness pattern generated with a Gaussian spectral density $S(q) = \pi\Delta^2\Lambda^2 \exp(-q^2\Lambda^2/4)$ ($\Delta = 0.1$ nm, $\Lambda = 6$ nm). (a) Distribution of the roughness height $\delta(\mathbf{r})$ and (b) the local current density $j(\mathbf{r})$ are plotted.

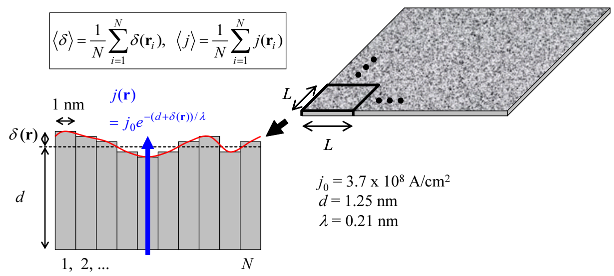

Fig. 2. Calculation method for the distribution of the tunneling current through the samples. (a) The samples with an area of $A = L^2$ are cut out from the generated roughness pattern, and (b) in each sample the average roughness height $\langle\delta\rangle$ and the average current density $\langle j\rangle$ are evaluated by discretizing the sample area into small sections ($\ll \Lambda^2$).

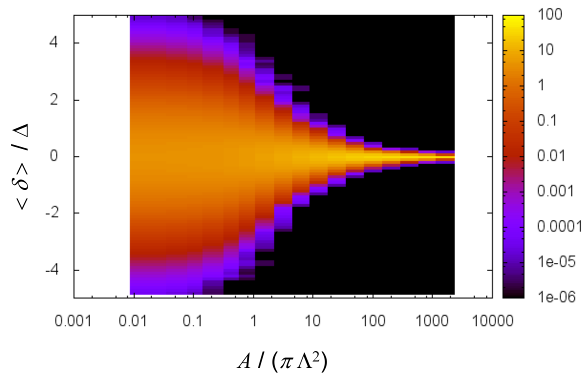

Fig. 3. Probability distributions of the average roughness height $\langle\delta\rangle$ simulated for various sample area A.

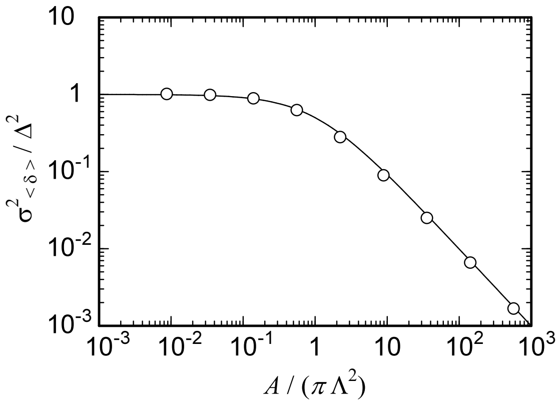

Fig. 4. Variance of $\langle\delta\rangle$ plotted as a function of A. Dots represent the numerical simulation results, which are well fitted by the analytical form given in Eq. (1) (solid line).

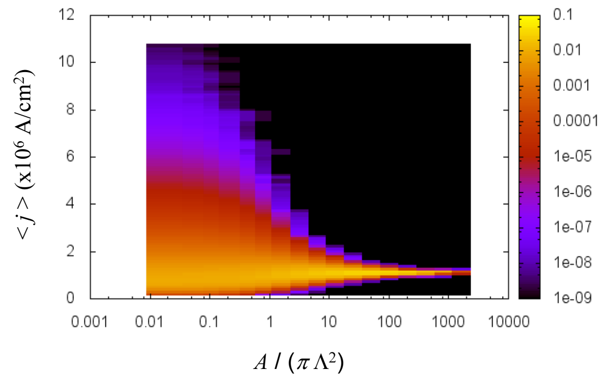

Fig. 5. Probability distributions of the average current density $\langle j\rangle$ simulated for various sample area A.

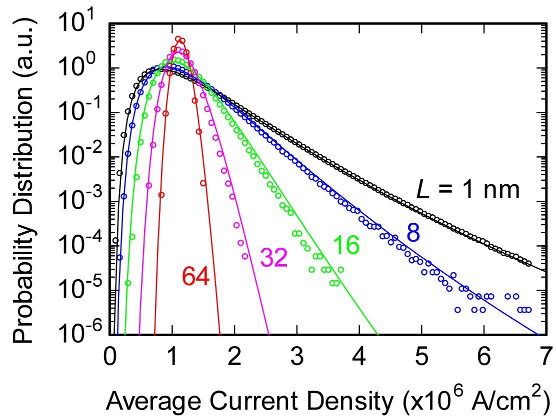

Fig. 6. Probability distributions of $\langle j\rangle$ simulated for various sample size L. Dots represent the numerical simulation results, and the solid lines are the analytically calculated results.