

Some Considerations on Conductance Fluctuations in Mesoscopic Structures

Bobo Liu, R. Akis, and D. K. Ferry

School of Electrical, Computer, and Energy Engineering, Arizona State University
Tempe, AZ 85287-5706 USA
e-mail: ferry@asu.edu

Universal conductance fluctuations have been observed in mesoscopic semiconductors for decades [1]. These fluctuations arise from the presence of a random potential in the semiconductor, which arises from e.g. impurities present in the material. The common theory suggests that the amplitude of these fluctuations will be a factor $\sqrt{2}$ smaller for magnetic field variation as compared to Fermi energy (or gate voltage) variation. This arises from breaking of time reversal symmetry in the magnetic field and a reduction in the diffuson channel in the quantum conductance. Within this small factor, it has been generally believed that a form of an “ergodic theorem” related the expected values for these two types of parameter sweep. Recently, experimental work on graphene has raised questions about such a theorem [2]. It was found that sweeps of gate voltage led to fluctuations that were more than a factor of 3 higher than sweeps of magnetic field.

Here, we explore Fermi sweep induced fluctuations versus magnetic field sweep induced fluctuations, to examine whether an ergodic theorem exists.

We use a recursive scattering matrix formulation to solve for the quantum transport through our active region, which is projected onto a discrete lattice in two dimensions. Here, the recursive approach follows that of Usuki [3] and our previous formulations. The use of a discrete lattice imposes a cosinusoidal band onto the eigenvalues of the slice Hamiltonian, and that allows us to study both the normal parabolic band behaviour and a quasi-linear energy behaviour near the center of the energy band. Thus, we can determine whether the observed effects in graphene [2] are unique to graphene or appear in any quasi-linear energy structure.

We find a range of results depending upon the precise structure and the level of the random potential. When a modest random potential is superimposed upon a weak quantum dot confinement, then we find for both parabolic and quasi-linear bands that the rms amplitude of the fluctuations is about 0.3 (in units of $2e^2/h$) for Fermi energy sweeps and about 0.28 for magnetic field sweeps. The former is close to the value of 0.36 expected from diagrammatic Green’s function theory [1]. If the quantum dot is removed, the magnetic sweeps give smaller amplitude of order 0.1. If the amplitude of the random potential is increased to a large value, little change is seen in the Fermi energy sweeps (other than a reduction in overall conductance), but the fluctuations in the magnetic sweeps are reduced further to about 0.07.

These results suggest that there is no universal relationship between the two types of sweep, particularly no $\sqrt{2}$ connection, as the ratio of the amplitudes for the two sweeps can be found to vary from near 1 to more than 4. This further suggests that the observations in graphene [2] may be as much due to very strong disorder as to the uniqueness of the graphene energy structure.

- [1] See, e.g., D. K. Ferry, S. M. Goodnick, and J. P. Bird, *Transport in Nanostructures*, 2nd Ed. (Cambridge Univ. Press, Cambridge, 2009) Ch. 7.
- [2] G. Bohra, R. Somphonsane, N. Aoki, Y. Ochiai, R. Akis, D. K. Ferry, and J. P. Bird, “Nonergodicity and microscopic symmetry breaking of the conductance fluctuations in disordered mesoscopic graphene,” *Phys. Rev. B* **86**, 405(R) (2012).
- [3] T. Usuki, M. Saito, M. Takatsu, R. A. Kiehl, and N. Yokoyama, “Numerical analysis of ballistic-electron transport in magnetic fields by using a quantum point contact and a quantum wire,” *Phys. Rev. B* **52**, 8244 (1995).

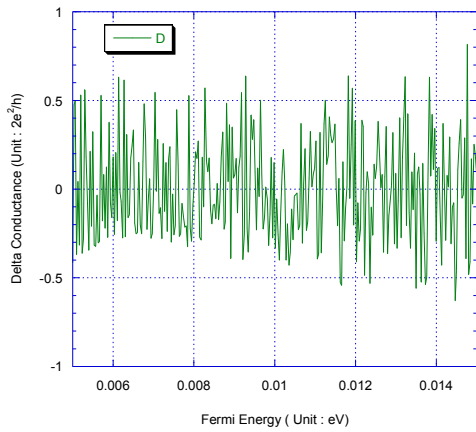


Fig. 1. Fluctuations for Fermi sweep in the parabolic regime. A small, soft walled quantum dot is also present.

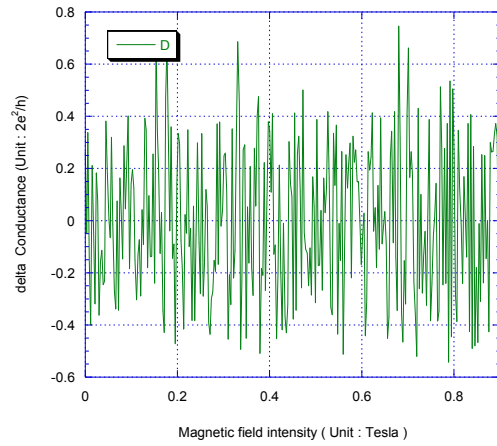


Fig. 2. Fluctuations for magnetic field sweep in the parabolic regime. A small, soft walled quantum dot is also present. The Fermi energy is 10 meV.

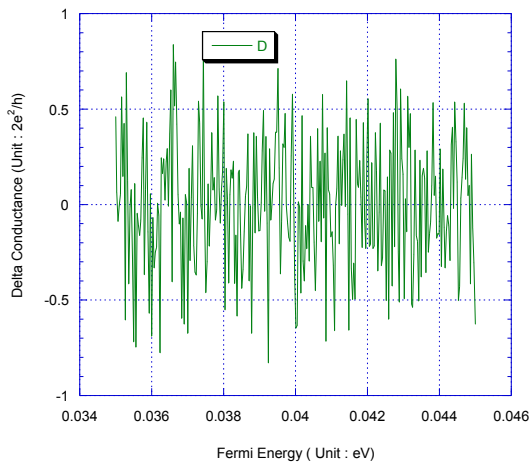


Fig. 3. Fluctuations for Fermi sweep in the quasi-linear regime. A small, soft walled quantum dot is also present.

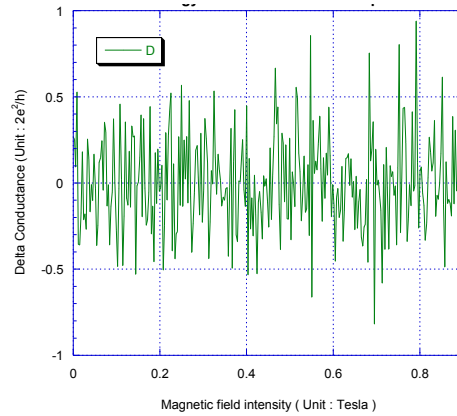


Fig. 5. Fluctuations for magnetic field sweep in the quasi-linear regime. A small, soft walled quantum dot is also present. The Fermi energy is 43 meV.

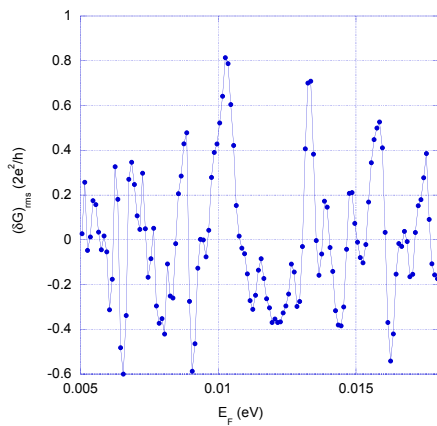


Fig. 2. Fluctuations for Fermi sweep in the parabolic regime, with no quantum dot potential, but with strong scattering.

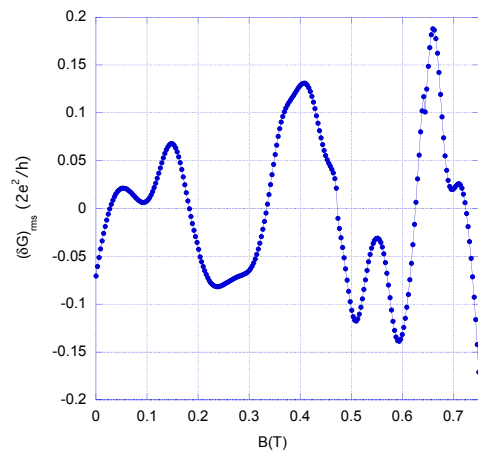


Fig. 5. Fluctuations for magnetic field sweep in quasi-linear regime for strong scattering. The Fermi energy is 58 meV