Numerical simulations of scanning gate spectroscopy on bilayer graphene in the Quantum Hall regime

D. Logoteta*, P. Marconcini*, M. R. Connolly†, C. G. Smith†, and M. Macucci*
* Dipartimento di Ingegneria dell'Informazione, Università di Pisa, Via G. Caruso 16, I-56122 Pisa, Italy
† Cavendish Laboratory, Department of Physics, University of Cambridge, Cambridge, CB3 0HE, UK
e-mail: demetrio.logoteta@for.unipi.it

A widely accepted theory models the Quantum Hall effect as a percolation phenomenon, in which conduction can only occur along paths connecting localized states [1]. In particular, in the presence of an underlying potential with sufficiently slowly spatial variations, the electronic states can be visualized as confined in equipotential strips surrounding hills and valleys of the potential and getting close to each other at the saddle points of the potential, where tunnelling can take place. The nature of the effect makes scanning gate spectroscopy (SGS, see Fig. 1) a particularly suitable experimental technique, due to the possibility to map the effect on transport of perturbations of the connections between localized states, which are expected to correspond to large variations ("hotspots") of the longitudinal conductance. Here we focus on numerical simulations of SGS experiments on bilayer graphene in the neighborhood of a Quantum Hall transition, trying to correlate transport effects with some details of the potential which are not directly accessible to experiments. We adopt the random network model proposed by Chalker and Coddington [2], representing the bilayer sample as a regular network, with meshes corresponding to the current loops and nodes corresponding to the saddle points of the potential (Fig. 2). The randomness in the relative positions of the saddle points is accounted for by randomizing the phase shift relative to the links connecting the nodes. The transmission properties of each node can be varied by tuning the value of a dimensionless quantity Θ, a parameter defining the associated 2×2 scattering matrix. The parameter Θ is assumed to be a nondecreasing function of the difference between the energy associated with the motion of the guiding center of the incident electron and the potential at the saddle point. The value of the potential associated to each node is decomposed into two components: the value in the absence of back-gate and probe bias, and the perturbation due to biasing these electrodes. The first component is randomly extracted, in order to account for the fluctuations of a realistic potential landscape, while the second is calculated by equating the electrostatic and the quantum expressions for the induced perturbation of the local charge density. The calculation of the conductance is performed in the Landauer-Büttiker framework, evaluating the transmission matrix with a recursive scattering matrix approach. In order to simulate SGS maps we scan the tip over each node of the network, calculating, for each tip position, the variation of conductance with respect to the condition of zero probe bias; the results are then plotted as a function of node position. The reported results, which are in good agreement with the experimental measurements in [3], refer to the simulation of a 2 μm $\times$ 4 μm flake, modeled with a network of 38 $\times$ 76 nodes; the considered potential fluctuations are uniformly distributed within an interval of amplitude $\approx$ 10 meV. In Fig. 3 we report the percolative riser of the conductance obtained by increasing the back-gate voltage (V_{BG}); in Fig. 4 we show three maps of the absolute value of the current density, corresponding to values of V_{BG} located before, at, and after the percolative threshold. Finally, in Figs. 5 and 6 we show a sequence of simulated SGS images for different values of V_{BG} and of the probe voltage, respectively.

REFERENCES

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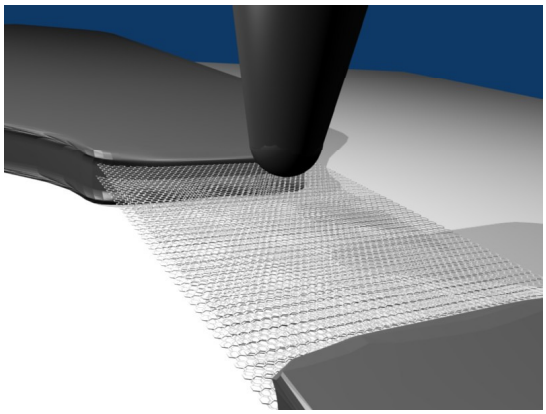

Fig. 1. Artist's rendering of a bilayer graphene flake scanned by a probe.

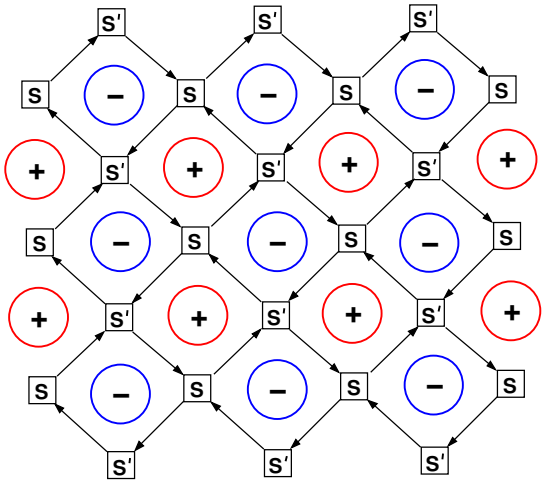

Fig. 2. Sketch of the adopted network model: the arrows describe the guiding center motion around the hills (+) and the valleys (-) of the potential, while the scattering matrices S and S' (which differ in the dependence on Θ) characterize the tunnelling at the saddle points. Adapted from [1].

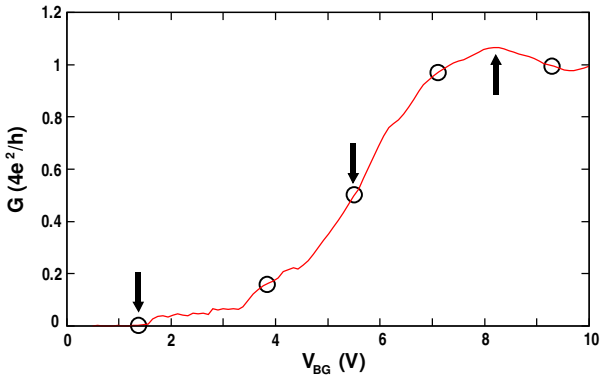

Fig. 3. Plot of the conductance of the considered network as a function of V_{BG}.

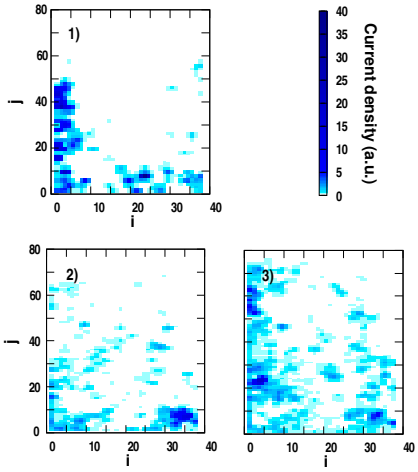

Fig. 4. Maps of the absolute value of the current density for the values of V_{BG} indicated by arrows in Fig. 2 (the images are ordered for increasing values of V_{BG}); the current injection is from the lower side of the images. The scales on the axes refer to node numbers.

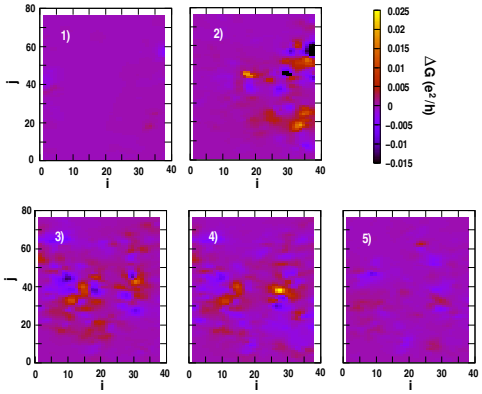

Fig. 5. Sequence of maps of the variation of the conductance for the values of V_{BG} indicated with circles in Fig. 2 (the images are ordered for increasing values of V_{BG}). The probe voltage is held at 1 V.

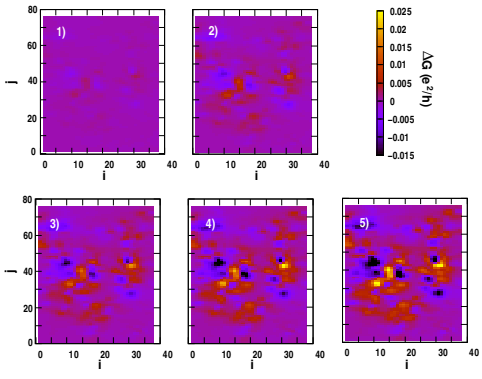

Fig. 6. Sequence of maps of the variation of the conductance for five uniformly spaced values of the probe voltage, from -1 V to 1 V. The back-gate voltage is held at 5.6 V.