# Universal Transport Properties of Random Nanowires

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Semiconductor nanowires (NWs) offer plenty of opportunities in quasi-one-dimensional physics and can also be considered to be promising candidates for nanoelectronics. In practical applications, sample-to-sample variability becomes a crucial issue necessitating statistical study of conductance properties. A standard approach to study a disordered wire is to introduce an extra random potential $\delta V$. Recently developed equivalent model (EM) offers an effective way to perform such studies in realistic NWs [1]. Here we present the results on universal transport properties of random NWs.

We consider a one-particle Hamiltonian

$$[h_0 + \delta V]|n\rangle\langle n| + [w|n\rangle\langle n+1| + \text{c.c.}]\,, \tag{1}$$

where $|n\rangle$ represents the $N_{\text{EM}}$-dimensional EM basis in the $n$-th unit-structure of a NW and $h_0$, $w$ define the basis-transformed Hamiltonian in a regular wire (Fig. 1). Given atomistic wire, the EM method builds an equivalent quantum chain of low dimension $N_{\text{EM}}$ with the same transport characteristics [2]. Figure 1 shows an example of the EM used in the simulations. Since the EM basis is delocalized within each unit-structure, the statistical invariance condition implies $\delta V$ in the random-matrix form

$$\begin{aligned}&\langle \delta V_{\nu\mu}\rangle = 0,\\ &\langle \delta V_{\nu\mu}\delta V_{\nu'\mu'}\rangle = \alpha^2(\delta_{\nu\nu'}\delta_{\mu\mu'} + \delta_{\nu\mu'}\delta_{\mu\nu'}),\end{aligned} \tag{2}$$

where $\nu$, $\mu$ numerates the EM basis states and $\alpha$ can be estimated from the microscopic Hamiltonian.

The physical characteristics of the model Eqs. (1), (2) are the localization length $\lambda(E,\alpha)$. The R-matrix propagation technique [3] is used to evaluate two boundary blocks $G_{1N}$ and $G_{NN}$ of the Green's function in a closed system of $N$ unit-structures. $\lambda(E,\alpha)$ can be calculated from the eigenvalues of $G_{1N}G_{NN}^{-1}$ in the limit of large $N$ (see Fig. 2). Figure 3 presents the computed localization length in a wide range of parameter $\alpha$. The $E$-dependence at $\alpha = 0.001$ eV is found to correlate with transport properties of the original tight-binding model with surface roughness without strain.

To establish a relation between the localization length and the conductance properties, we have calculated statistics of the transmission function within the same $\alpha$-interval. Figure 4 shows an example of the averaged $\overline{T}$ and the statistical variation $\overline{T^2} - \overline{T}^2$ at $\alpha = 0.02$ eV. The horizontal plane indicates the maximum dispersion at $[\overline{T^2} - \overline{T}^2]_{\max} \approx 2/15$ consistent with the classical result for the universal conductance fluctuations [4]. Detailed study exhibits similar $L$-dependence at all $\lambda$s for $N_{\text{ch}} > 1$ open channels (Fig. 5). In particular, we have observed the weak localization regime up to $L \approx \lambda$ (Thouless criterion) where the statistical dispersion reaches the classical value. This findings suggest applicability of the macroscopic description to the atomistic transport in realistic NWs which may have impact on statistical modeling of NW MOSFETs.

## References

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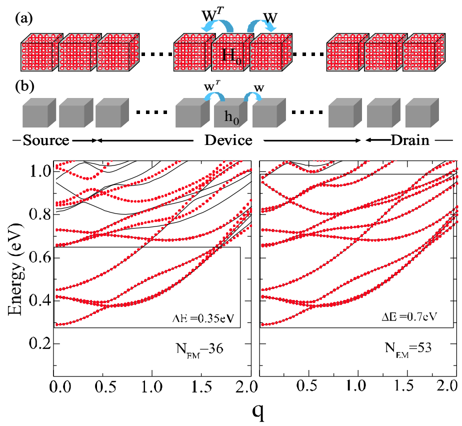


Fig. 1. EM method: The tight-binding picture (a) is transformed to a low-$D$ equivalent chain (b). Band structure in two EMs ($N_{\mathrm{EM}}$ = 36, 53) for a thin p-Si NW used in the simulations. $\Delta E$ is the energy window reproduced by the EM.

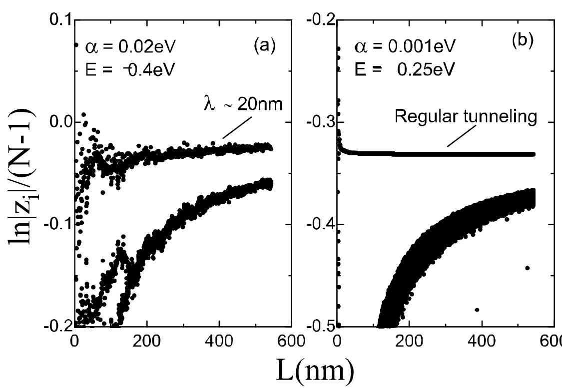


Fig. 2. Asymptotic behaviour of the eigenvalues $z_i$ of $G_{1N}G_{NN}^{-1}$. The minimum asymptotic value determines the localization length $\lambda$ in the system (a) which is to be compared with the ordinary tunneling at small $\alpha$ (b).

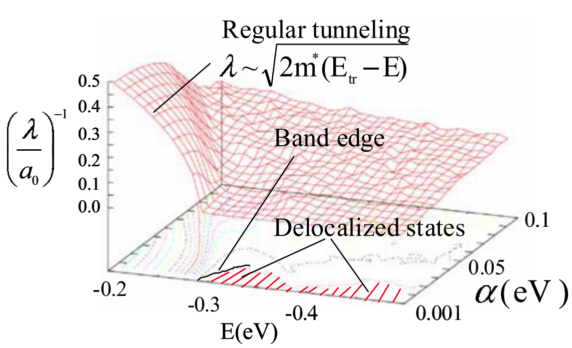


Fig. 3. The localization length $\lambda(E,\alpha)$. Contour plot illustrates enhancement of the ordinary tunneling. Shadow area represents delocalized states in the system.

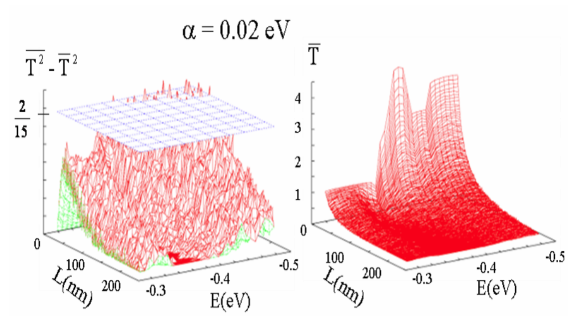


Fig. 4. $\bar{T}(E,L)$ and $\mathrm{var}T(E,L)$ at fixed $\alpha$. The horizontal plane corresponds to the classical result for the conductance fluctuations.

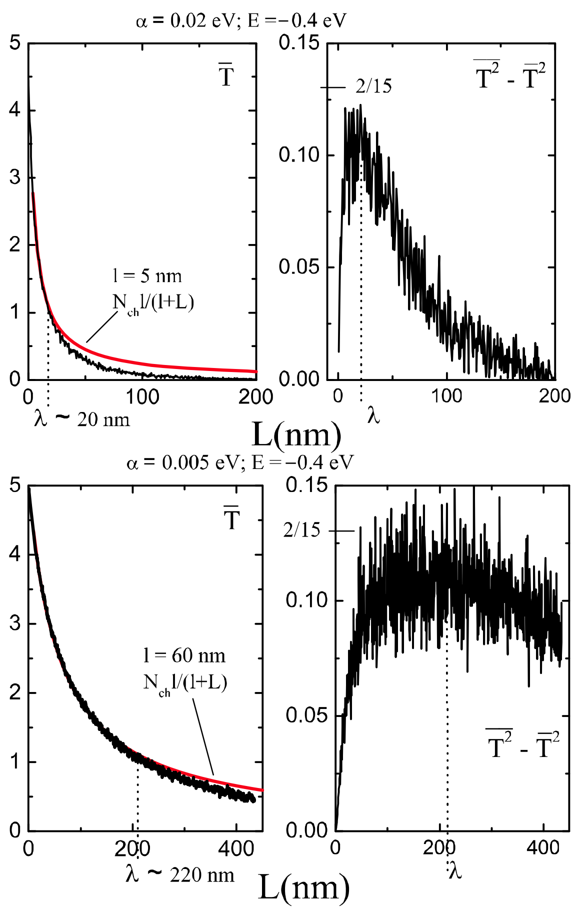


Fig. 5. Universal behaviour of the transmission function. The Thouless condition $\bar{T}(\lambda) = 1$ and the maximum statistical dispersion $\approx 2/15$ at $\lambda = 20$ nm (upper panel) and $\lambda = 220$ nm (low panel).