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Indirect Optimal Control of a Qubit

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An indirect optimization scheme for the dynamic control of open quantum system is presented. It is demonstrated at the example of a spin 1/2 system which strongly couples to a bath of phonons. It reveals the possibility of control of the effective system-bath coupling, provided that the system is addressed in the quantum regime. The method is compared to direct algorithms, such as the genetic algorithm.

INTRODUCTION

The fundamental principle of quantum interference between competing interactions has been seen as a potential principle of operation for electronic and electro–optic nanoscale devices of the future. Unlike optical interference effects, electronic or spin–based quantum interference effects in solids are generally difficult to establish and to maintain.[1] The fundamental reason is the generally strong coupling of of the electron to its environment - other electrons and lattice ions. Nevertheless, due to the well developed semiconductor industry, there has been considerable research effort in the development of electron (electron spin) based quantum devices, such as single–electron devices, resonant-tunneling-based devices, and qubits based on the spin degree of freedom or point defects with a discrete spectrum in the main energy gap.

A key issue in ones ability to dynamically control (elementary) quantum systems along a desired quantum trajectory is the control of its interaction with the environment. Ideally one would wish to be able to eliminate the system–environment interaction altogether. Next best would be to minimize loss of coherence. This leads us directly to an optimization problem (inverse problem) in which optimal control fields are sought which, in general, stabilize coherence of a quantum system. A related task is the optimization of control fields to maximize induced quantum interference effects. Best results can be expected when one is able to address the system on a time scale where its full quantum nature can be utilized.

THE INDIRECT METHOD

In this contribution we present an indirect optimal control scheme for dissipative quantum systems which we apply to a study of a qubit which couples strongly to a phonon bath. For this purpose, the physical objective is formulated within a cost functional $J(\varepsilon, \rho, t)$ where $\rho(t)$ is the density operator of the (sub) system obeying a general kinetic equation of the form

$$\dot{\rho}(t) = \int_0^t dt' K(t, t'; \rho(t'), \varepsilon(t')), \quad \rho(0) = \rho_o,$$

for $t \in [0, T[$, whereby the kernel $K(t, t'; \rho(t'), \varepsilon(t'))$ depends on the history of the system $\rho(t')$, as well as the real–valued external control $\varepsilon(t')$ ($\in \mathcal{L}^2[0, T]$) for $t' \leq t$, whereby causality is observed. This kinetic equation serves as a holonomic constraint to the cost functional in form of a non–Markovian set of differential equations in the density matrix elements. The cost function may be used to drive the system from a given initial state to a given final state, to trap it in a quantum state, or to optimize absorption, photo–current yield, etc. An optimum control field is one which minimizes the cost functional, whereby $\rho(t)$ plays the role of a dependent variable.

Here, this inverse problem is solved using an indirect method based on the concept of a co–state.[2] The latter may be viewed as a time–dependent Lagrangean multiplier which is used to eliminate variations with respect to the dependent variable

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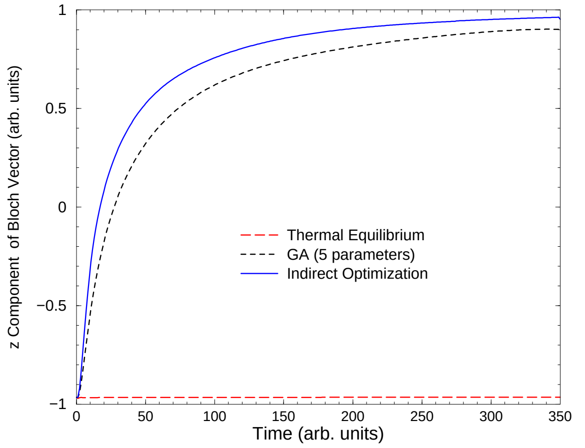

Fig. 1. Driving a qubit from its ground state into the "up" state at given target time: long dashed line: ground state (control field=0); dotted line: genetic algorithm (Gaussian pulse); solid line: indirect method.

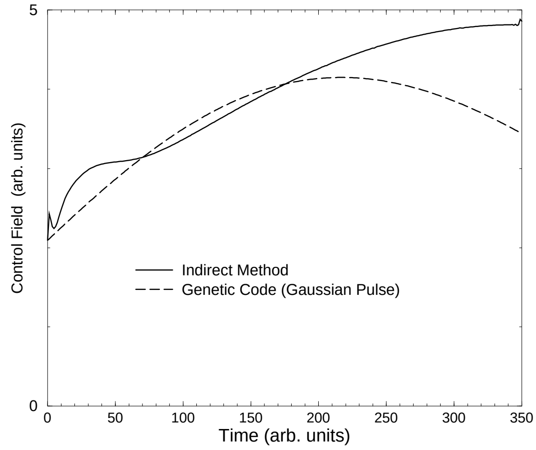

Fig. 2. Control field for driving a qubit from its ground state into the "up" state at given target time: solid line: indirect method; dotted line: genetic algorithm (Gaussian pulse).

$\rho(t)$, or, as the canonical momentum associated with the variable ρ when following the Euler–Hamilton variation principle.[3] This procedure leads to a generalization of Hamilton's equations of motion to non–Markovian systems.

Motivated by the current interest in the control of qubits, we demonstrate this approach for a qubit (spin-1/2 system, two–level system) which is coupled to a bath of phonons within a driven spin–boson model with strong electron–phonon coupling.[4], [6] When one is able to address the system on a time scale where it reveals its non–Markovian quantum nature, there is generally a better chance to control the system–bath interaction than when one is in a time regime where the latter behaves classically.[5] The difference can be attributed to quantum interference.

As an example, Fig. 1 gives the results for driving a qubit from its thermal ground state ($z \approx -0.96$) to z=1 at target time 350 (arb. units). This task is challenging since the target time is short compared to the characteristic relaxation time of the system. For comparison, the result for a Gaussian pulse which was optimized using a genetic code is given also. Fig. 2 gives the selected control fields for the two cases. Details will be presented in the full paper.

Acknowledgment

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