Control of Fano Resonances and the Transmission Phase of a Multi-Terminal Aharanov-Bohm Ring with Three Embedded Quantum Dots

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In order to fully characterize the transport properties of mesoscopic electronic devices, such as quantum dots (QD's) and rings, information on both the transmission coefficient (obtained experimentally from the conductance) and the phase are necessary. Phase measurements of the transmission can in principle be obtained through the interference effects produced in an Aharonov-Bohm (AB) ring with one or more embedded QD's. Transmission through the QD has been shown to be coherent, maintaining the phase information of the electron wave, through the observed oscillatory AB-effect which is periodic with the flux quantum, Φ_0=h/e.

In a two-terminal device, current conservation and time-reversal symmetry require that the transmission is of the form, $T_{LR}(\Phi_0)=T_{LR}(-\Phi_0)$, which forces the phase of the AB oscillations to be either 0 or π . An abrupt jump in phase of magnitude π is seen at the maxima of the resonant transmission peaks. In a multi-terminal device, however, this phase rigidity is relaxed, and it becomes possible to determine the intrinsic phase shift produced by the QD by measuring the phase of the AB-oscillations at different energy values across the transmission peak.[1]

In this work, we study a four-terminal AB-ring with three QD's embedded in one arm (Fig. 1). A tight-binding model is employed to analytically obtain the transmission through the system. Results show that the magnitude and sharpness of the resonant peak phase-jumps diminish as a function of the degree of coupling to the extra terminals. Fano resonances, produced by interference between the resonant states of the QD's and the continuum path through the reference arm of the AB-ring, are shown to have their zeros lift off the real energy axis into the complex energy plane by opening the transmission through the ring, with coupling to the extra terminals by allowing V_d (see Fig. 1) to be non-zero. A simple analytical model of the Fano resonances shows naturally how the phase transition across the resonance peak will soften in the case where the transmission zero energy is no longer real, but complex.

Contour plots of the transmission in the complex energy plane also show that for this device with three embedded QD's it is possible to vary the coupling between the dots to obtain either three Fano dipole pairs, or to merge two of the dipoles into a Fano quadrupole (see Fig.'s 2 and 3). The Fano quadrupole behaves as a coupled object in which the two transmission peaks are preserved, but the zeros first merge and then move into the complex energy plane. As a function of magnetic flux through the ring, the zeros of the quadrupole orbit through the complex energy plane around the pair of poles corresponding to their transmission peaks.[2] With this device, the unique behavior of both the Fano quadrupole and the Fano dipole are exhibited simultaneously for proper ranges of the coupling parameters.

The enhanced degrees of control available with such a device give new possibilities for controlling and investigating the transmission and phase as a function of magnetic flux and inter-dot coupling.

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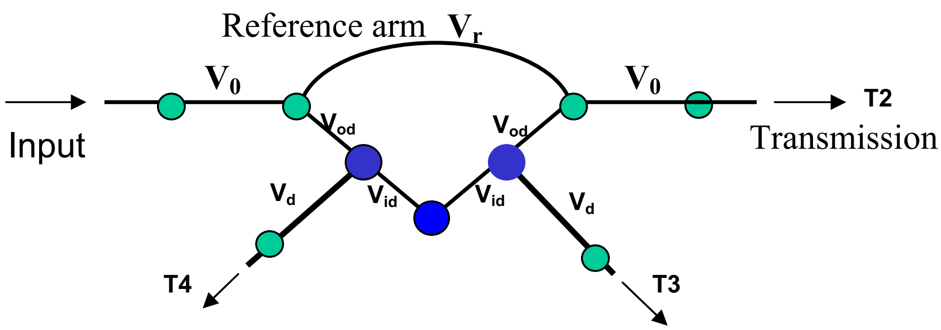

Fig. 1. Schematic of the four-terminal AB-ring with three embedded QD's in lower arm.

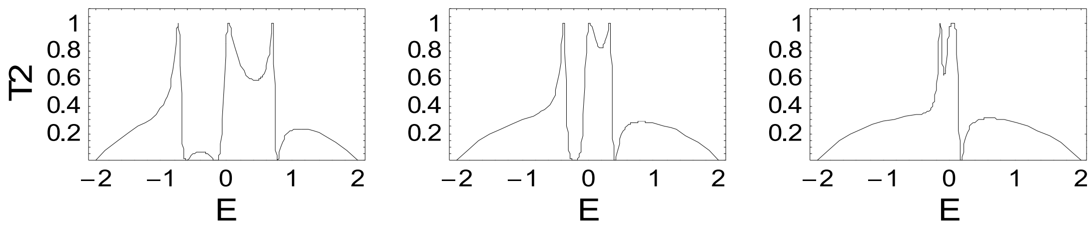

Fig. 2. Transmission plots showing the merging of the Fano zeros as inter-dot coupling, V_{id}, is decreased. V_{id} = 0.5, 0.25, and 0.10 (left to right). Coupling between outer dots and ring, V_{od}=0.3; coupling through reference arm, V_r=0.3, and V_0=1.0 in the leads.

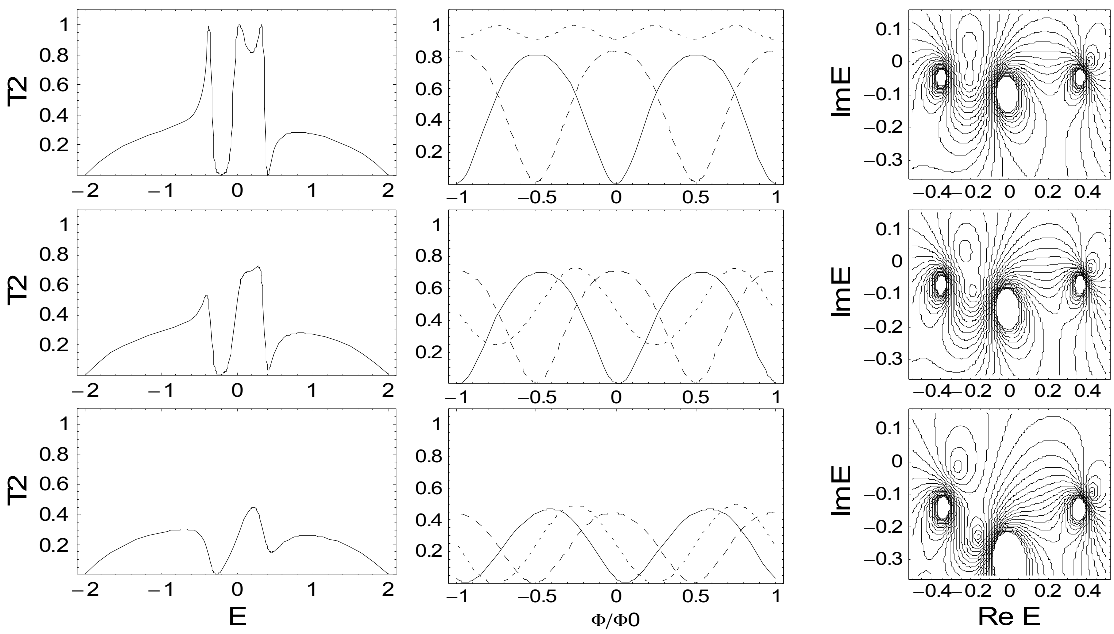

Fig. 3. Transmission vs. energy (Φ/Φ_0=0) and flux (E = -0.2, 0, 0.25, for solid, dotted, and dashed lines, respectively), and transmission contour plots showing Fano zero-pole pairs, for varying degrees of openness of the ring: V_d=0, 0.2, and 0.4 (top to bottom).