Wigner function for identical particles

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In the last decades it has become clear that a rigorous quantum approach to electron transport in nano-device design is necessary, not only for the solution of problems where quantum effects are essential but also for testing the semiclassical approximation by comparison with exact quantum results obtained for the very same system. The Wigner-function approach seems to be the most appropriate method to deal with space dependent problems since it explicitely refers to variables defined in an (**r**,**p**) Wigner "phase space".

Since its first definition in 1932 [1], the Wigner function was introduced for many-particle systems. Nevertheless, its main use in quantum transport physics has been almost exclusively confined to its single-particle form, since the physical quantity of interest usually is the current due to independent electrons.

The aim of this work is to extend the Wigner-function approach to quantum transport developed for the single electron case to a more complicated system of n indistinguishable particles. In particular, we study how the Monte Carlo technique and the Wigner paths method [2] can be applied to a single-particle Wigner function defined for a system of n interacting particles. Starting from the definition of the one particle Wigner function as the Fourier transform of the two point $G^<$ Green function [3], it is possible to extend the Wigner approach to the case where n identical interacting particles are considered.

First we apply this approach to the special case of two indistinguishable and non interacting particles, either bosons or fermions. The comparison of the results with the ones obtained for the case of two distinguishable particles, allows to clearly identify the effect in the Wigner representation of Pauli's exclusion principle (see Fig. 1).

Then we evaluate the time evolution of the interacting case. We succed in calcoulating the first pico-seconds of the time evolution of the ballistic term and of the first-order scattering term of the Coulonb interaction between two identical fermions. (see Fig. 2)

Once the indistinguishability condition and the electron-electron interaction have been rigourously inserted into the theory, it is possible to turn to the calculation of the classical limit and/or to the Hartree-Fock approximation, to have a better insight on the corrections that one has to introduce on the standard Monte Carlo calculation, in order to account for particle identity in electron-electron interaction.

[1] E. Wigner, Phys. Rev. **40**, 749 (1932)
[2] C. Jacoboni et al., Rep. Prog. Phys. **67**, 1033-1071 (2004)
[3] G.D. Mahan, Phys. Rep. (R. S. of Physics Letters) **5**, 251 (1987)

A full journal publication of this work will be published in the Journal of Computational Electronics.

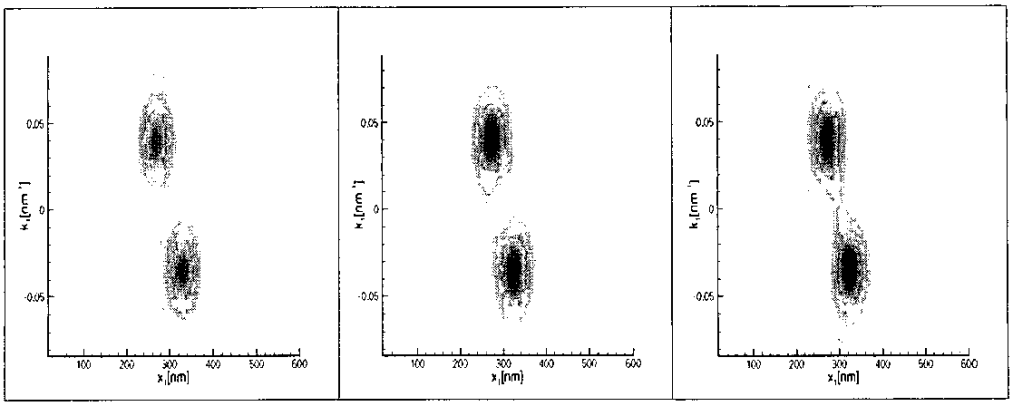

Fig. 1 Here we show the 2D Wigner function representation for wavefunctions given by minimum uncertainity wave packets: (A) the case of two distinguishable particles, (B) the case of two indistinguishable fermions and (C) the case of two indistinguishable bosons. The correlation due to particle identity can be seen in (B) and (C).

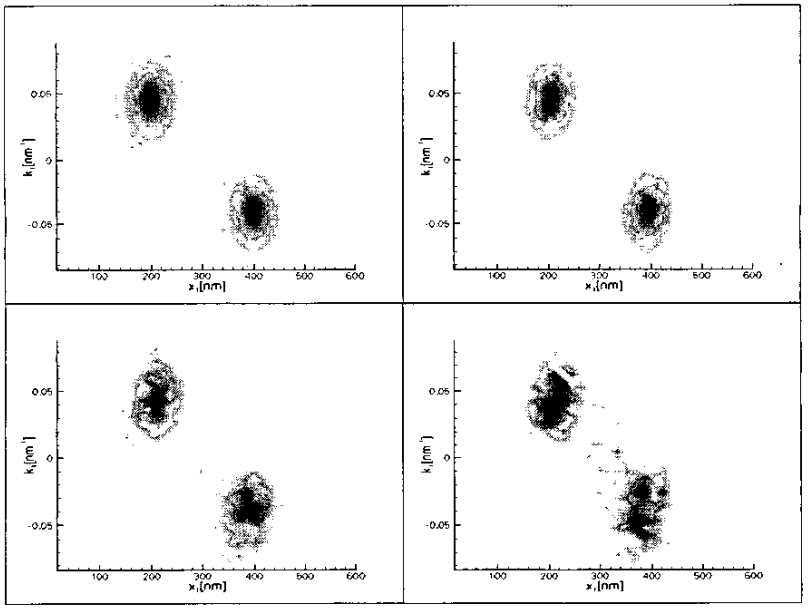

Fig. 2 The time evolution from 0 to 9 ps with the electron-electron interaction for wavefunctions given by minimum uncertainity wave packets for two indistinguishable fermions. The wave packets are retarded by e-e interaction.

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